Workshop and Summer School: From Statistical Mechanics to Conformal and Quantum Field Theory

8 January - 8 February, 2007

Bulk and Boundary Form Factors in QFT (1)

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Institute for Theoretical Physics, Eötvös University, Budapest

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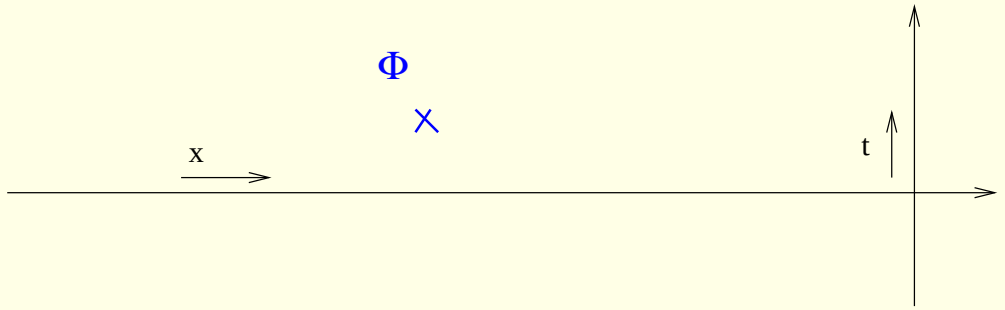

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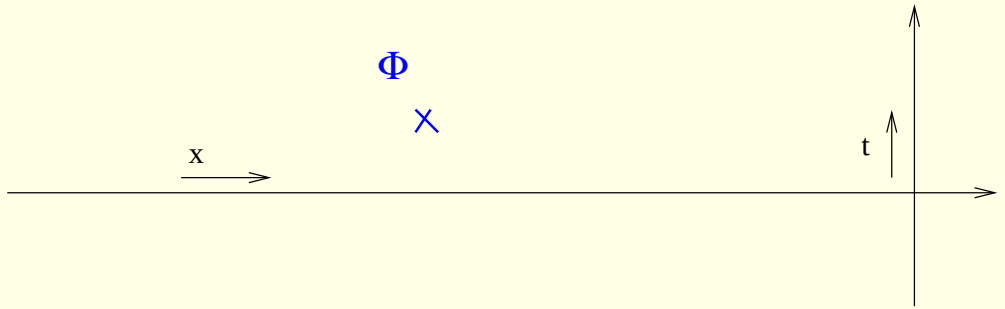

Boundary bootstrap programme for massive integrable boundary QFT

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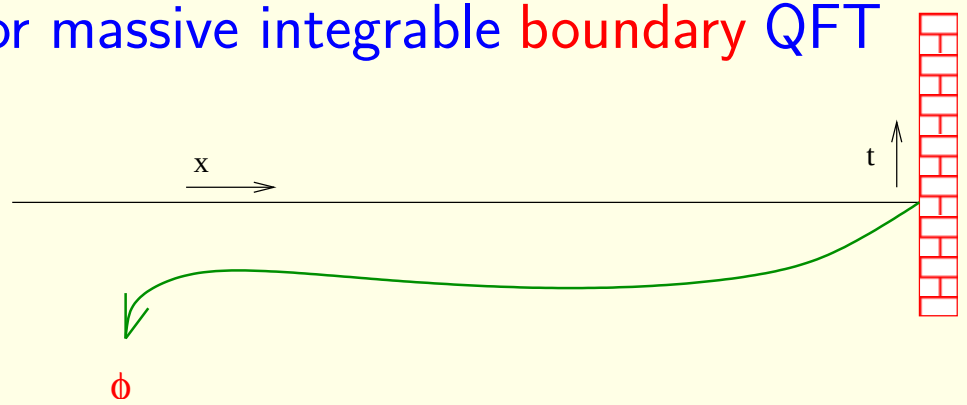

Definition of a QFT: Schemes
QFT={ Correlators of local operators}

$$\mathcal{L} = \frac{1}{2}(\partial_t \Phi)^2 - \frac{1}{2}(\partial_x \Phi)^2 - V(\Phi)$$

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↘

Path integral scheme ($t = -iy$)

operators: $\mathcal{O} = \partial^k \Phi \dots \partial^l \Phi$

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Schemes based on symmetries

↙

CFT scheme $V = 0$

spectrum from the symmetry

$H = L_0 + \bar{L}_0 = \frac{1}{2}(\partial_x \Phi)^2 + \frac{1}{2}(\partial_x \Phi)^2$

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Aim: explicit solution $\boxed{\mathcal{L} = \frac{1}{2}(\partial_t \Phi)^2 - \frac{1}{2}(\partial_x \Phi)^2 - \frac{1}{2}m^2\Phi^2}$ canonical quantization

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Requirements: $\Phi(x,t)$ satisfies:

$$(\partial_t^2 - \partial_x^2 + m^2)\Phi = 0 \quad ; \qquad [\Phi(x,t), \partial_t \Phi(x',t)] = i\delta(x-x')$$

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Maintain Poincare invariance: (implemented by the conserved charges)

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Conserved momentum $P = \int [\partial_t \Phi \partial_x \Phi]\, dx \quad ; \qquad \dot{P} = i[H,P] = 0$

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Energy-momentum tensor $\partial_\mu T^{\mu\nu} = 0$ and $H = \int T^{00} dx$; $\quad P = \int T^{01} dx$

Free scheme: solution

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Solution $\Phi(x,t) = \int \frac{dk}{2\pi\omega(k)} \left[e^{i\omega(k)t - ikx} a^{+}(k) + e^{-i\omega(k)t + ikx} a(k) \right]$

from equation of motion: $\omega(k) = \sqrt{k^2 + m^2}$ from commutation relation

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state	Energy	Momentum
vacuum $\|0\rangle$	0	0
one particle state $\|k\rangle$	$\omega(k)$	k
multiparticle state $\|k_1, \ldots, k_N\rangle$	$\omega(k_1) + \ldots \omega(k_N)$	$k_1 + \ldots + k_N$

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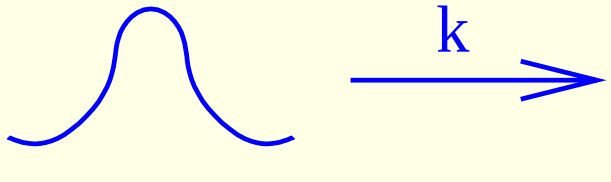
'localized' state $|f(k)\rangle = \int \frac{dk}{2\pi\omega(k)} f(k) a^+(k)|0\rangle$



Free scheme: correlators

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Local operators : $\partial_x^{n_1}\partial_t^{m_1}\Phi(x_1,t_1)\ldots\partial_x^{n_N}\partial_t^{m_M}\Phi(x_N,t_N)$:

Normal ordered products (creation operators are on the left)

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Normal ordered products (creation operators are on the left)

time ordered product

$$T(\Phi(x_1,t_1)\Phi(x_2,t_2)) \;=\; \begin{cases} \Phi(x_1,t_1)\Phi(x_2,t_2) & \text{if} \quad t_1 > t_2 \\ \Phi(x_2,t_2)\Phi(x_1,t_1) & \text{if} \quad t_2 > t_1 \end{cases}$$

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Normal ordered products (creation operators are on the left)

time ordered two point function

$$\langle 0|T(\Phi(x_1,t_1)\Phi(x_2,t_2))|0\rangle = \int \frac{dk_0dk_1}{(2\pi)^2}\frac{i}{k_0^2-k_1^2-m^2+i\epsilon}e^{ik_0(t_1-t_2)-ik_1(x_1-x_2)}$$

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Matrix elements of Φ

Using Poincare invariance: $\langle 0|\Phi(x,t)|k\rangle = \langle 0|e^{-iHt}\Phi(x,0)e^{iHt}|k\rangle = e^{i\omega(k)t-ikx}$

and so for $t_1 > t_2$

$$\langle 0|\Phi(x_1,t_1)\Phi(x_2,t_2)|0\rangle = \int \frac{dk}{2\pi\omega(k)}\langle 0|\Phi(x_1,t_1)|k\rangle\langle k|\Phi(x_2,t_2)|0\rangle$$

Wick theorem

Perturbative scheme

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Use pertrubation theory: $\mathcal{L} = \frac{1}{2}(\partial_t \Phi)^2 - \frac{1}{2}(\partial_x \Phi)^2 - \frac{1}{2}m^2\Phi^2 - \lambda U(\Phi) = \mathcal{L}_0 - \mathcal{L}_{pert}$

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Correlators: $\langle 0|T(\mathcal{O}_1 \dots \mathcal{O}_N)|0\rangle = \frac{{}_0\langle 0|T(\mathcal{O}_1 \dots \mathcal{O}_N e^{i\int H_{pert}})|0\rangle_0}{{}_0\langle 0|T e^{i\int H_{pert}}|0\rangle_0}$

$\langle 0|T(\mathcal{O}_1 \dots \mathcal{O}_N)|0\rangle = \sum_{n=0}^{\infty} \frac{(-i\lambda)^n}{n!} {}_0\langle 0|T(\mathcal{O}_1 \dots \mathcal{O}_N (\int U d^2x)^n)|0\rangle_0^{conn}$

for normal ordered perturbations all terms are finite

CFT scheme

CFT scheme

Hilbert space $\mathcal{H} = \sum_{(i,i)} \mathcal{V}_i \otimes \bar{\mathcal{V}}_i$ where $\mathcal{V}_i$ h.w. reps of the left chiral algebra

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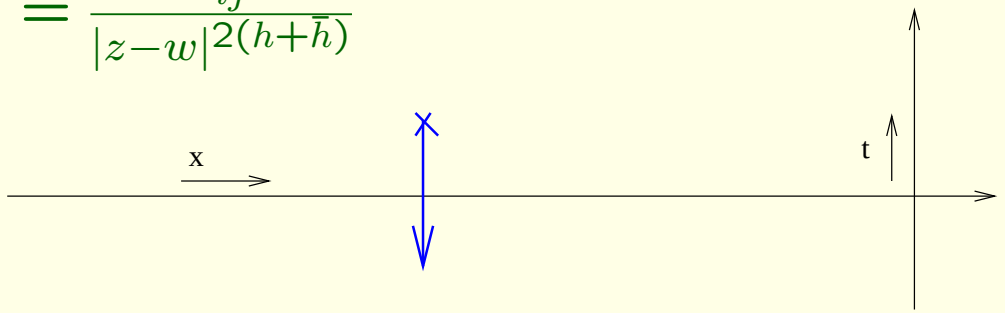

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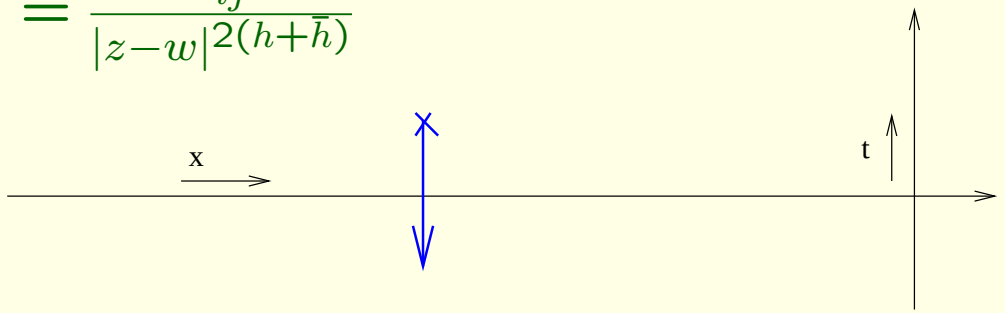

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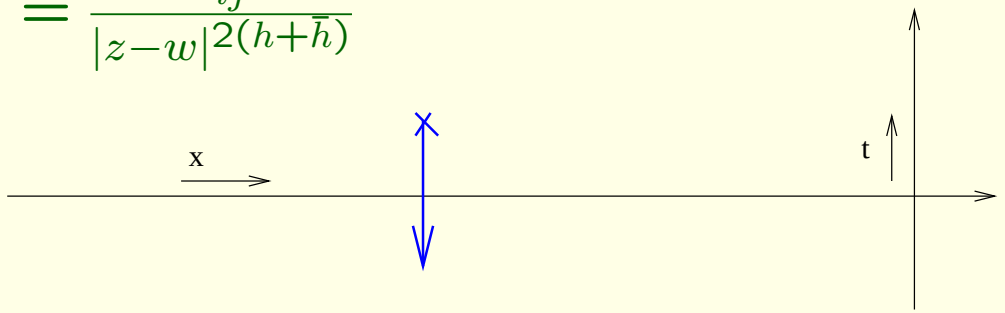

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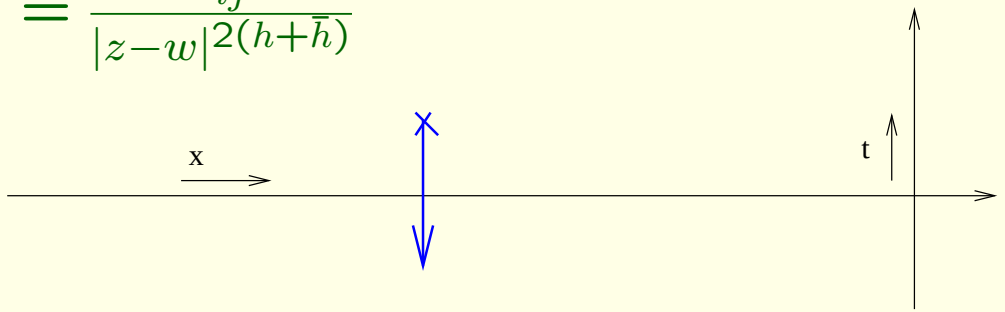

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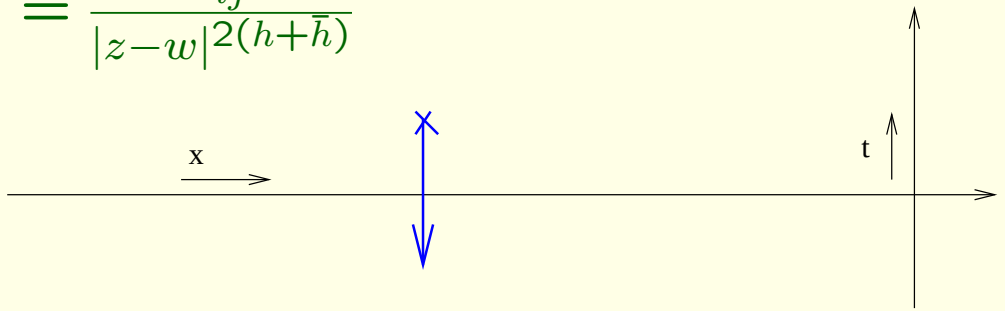

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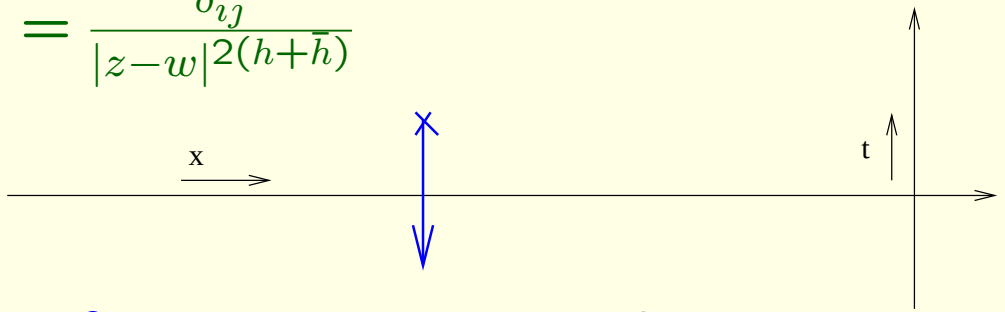

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Assumptions: spectrum, operator algebra smoothly changes

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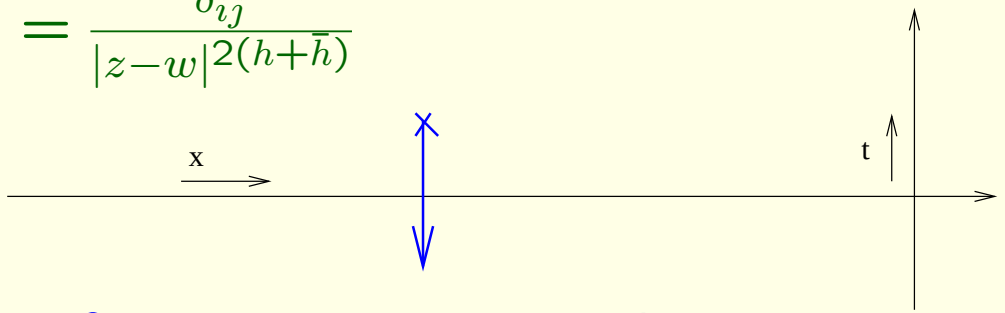

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Operator content
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notation $|k\rangle = |\theta\rangle$

$\boxed{\text{Higher spin conserved charges } E_n = q_n \cosh n\theta \text{ and } P_n = q_n \sinh n\theta}$

Integrable Field Theory scheme

$v_i = \sinh\theta_i$

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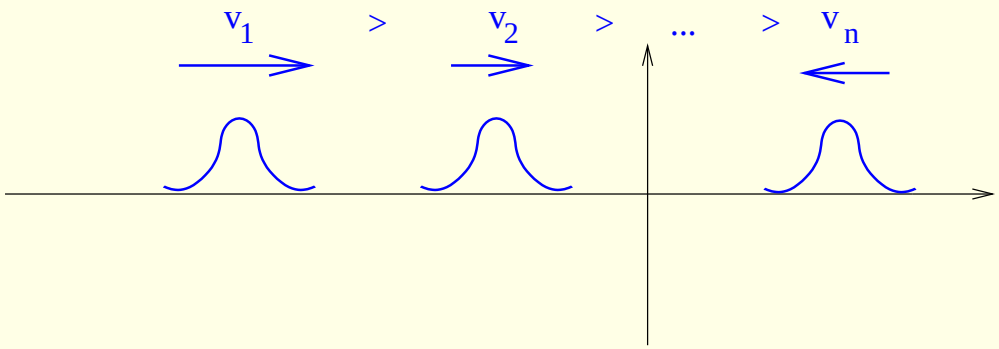
Bulk multiparticle state: with n particles



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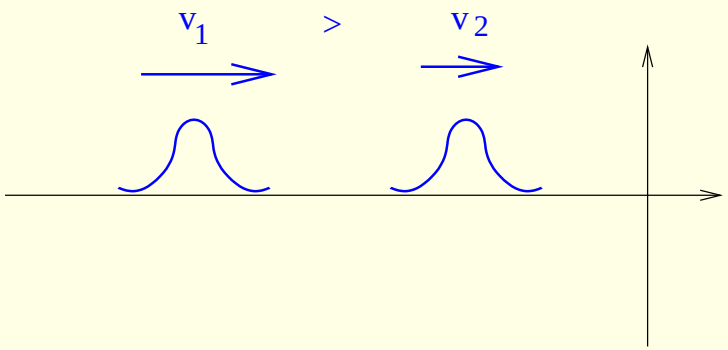
Bulk two particle state:



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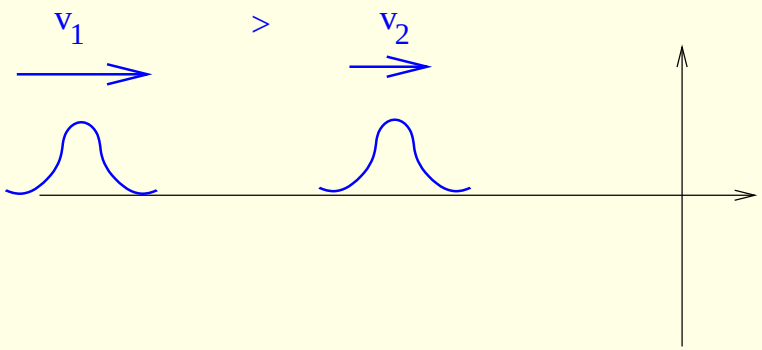
Bulk two particle in state: $t \longrightarrow -\infty$



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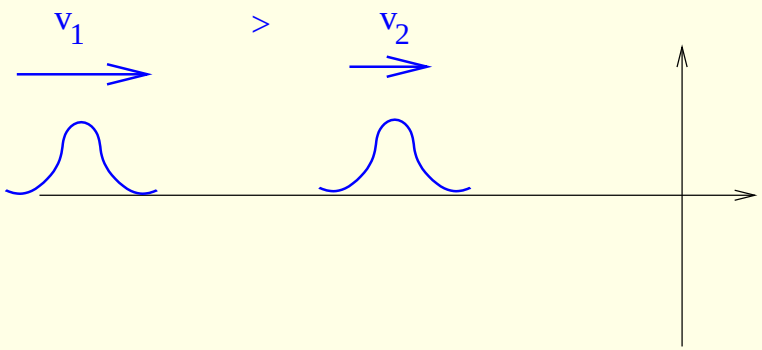
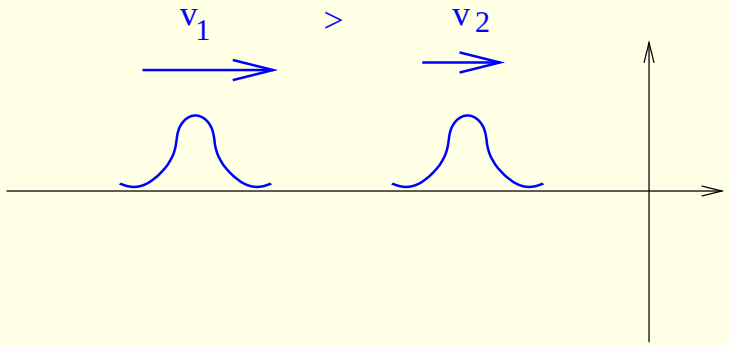


times develop



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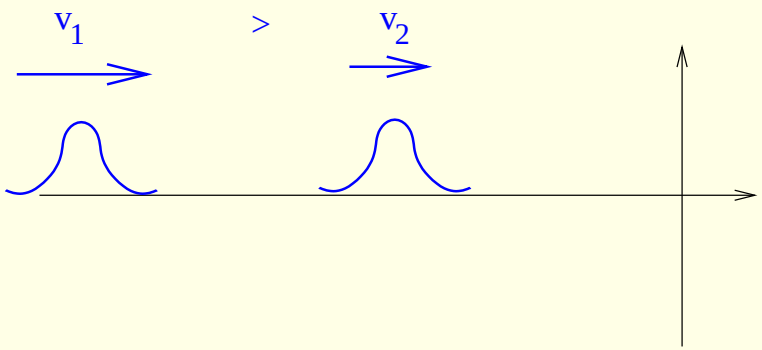
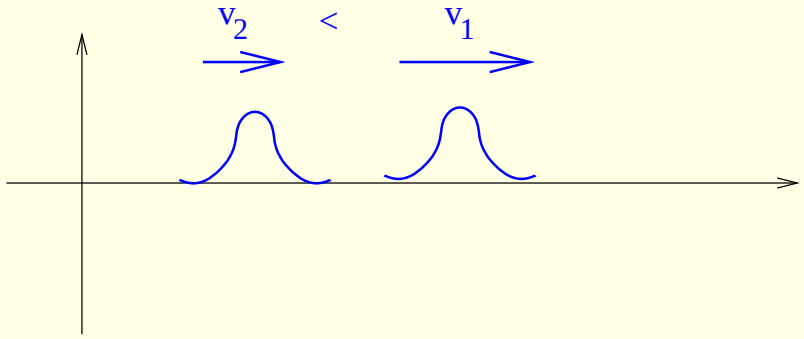


times develop further



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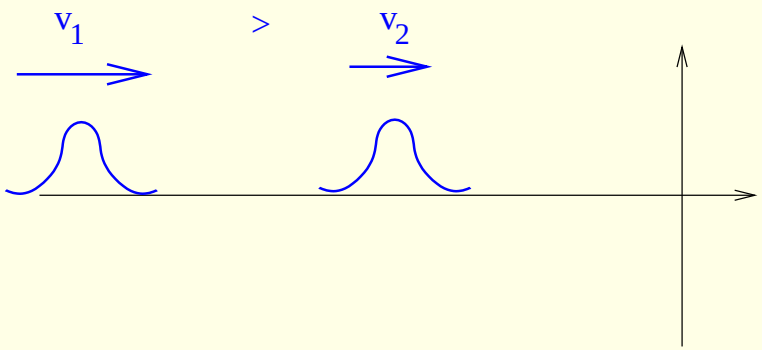
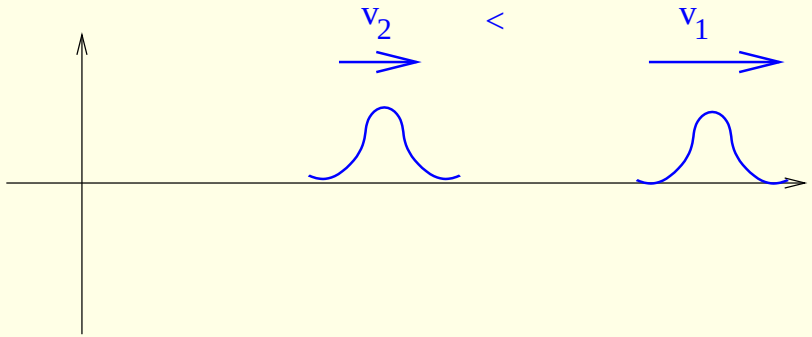


Bulk two particle out state: $t \to \infty$



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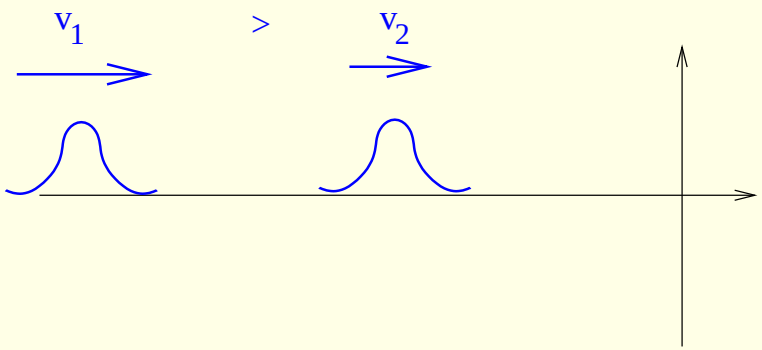

Free, noninteracting in particles

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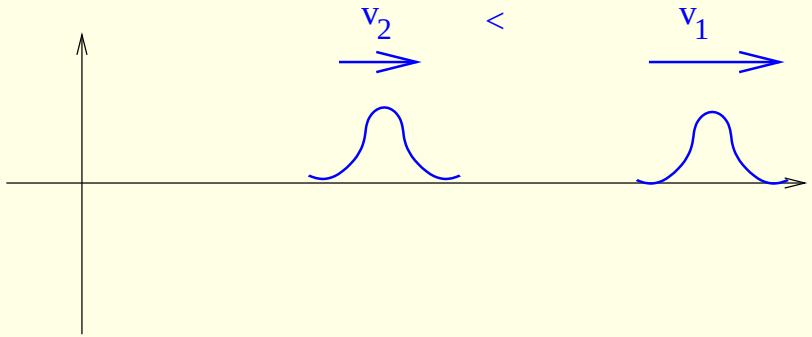

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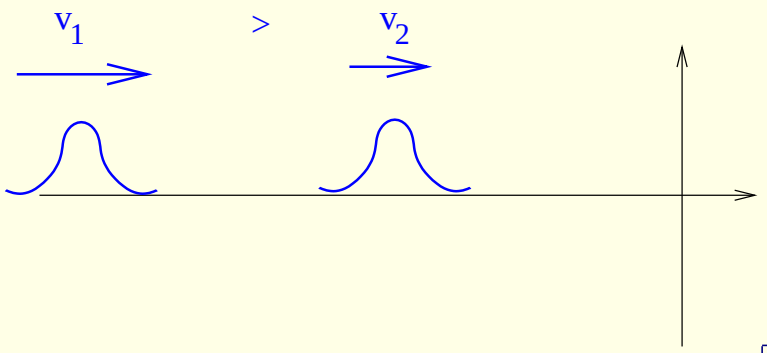

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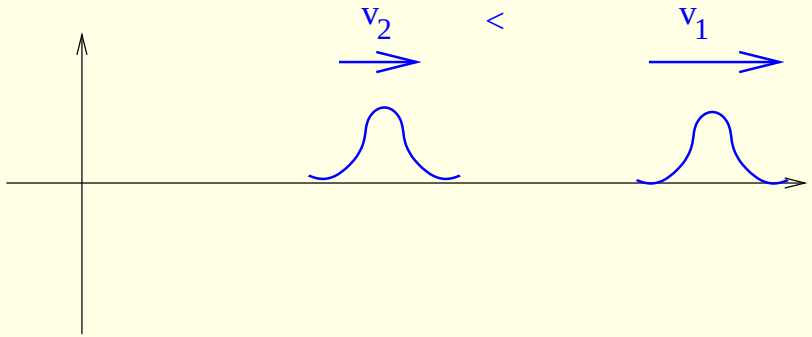

Free, noninteracting in particles $\leftarrow$ **S-matrix** $\rightarrow$ Free, noninteracting out particles

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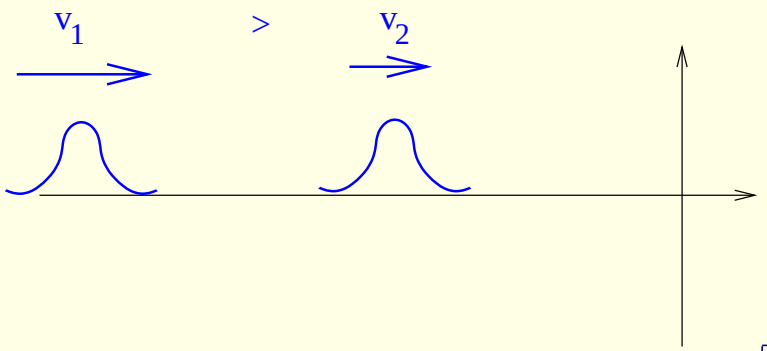

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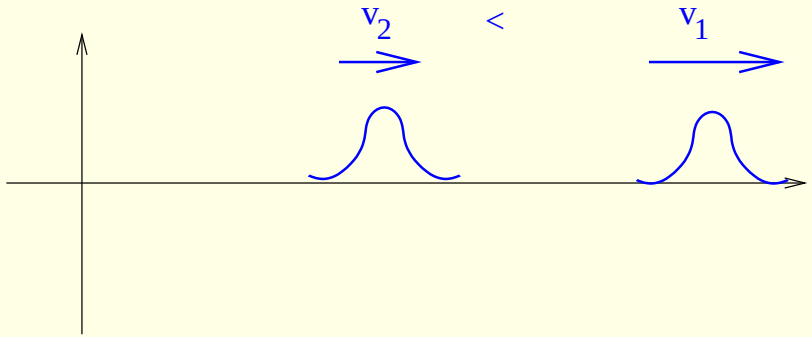

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$$|\theta_1, \theta_2\rangle^{in} = S(|\theta_1 - \theta_2|)|\theta_1, \theta_2\rangle^{out}$$

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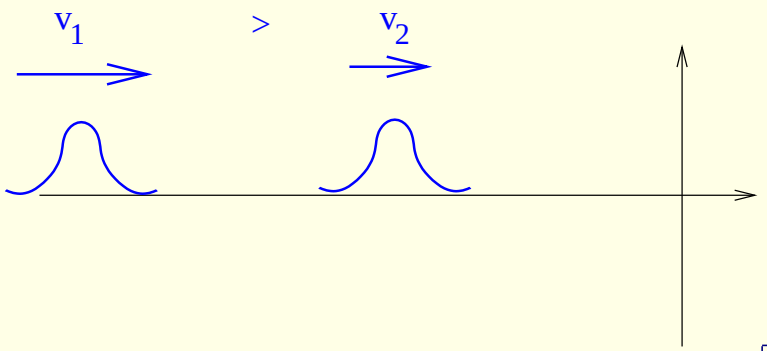

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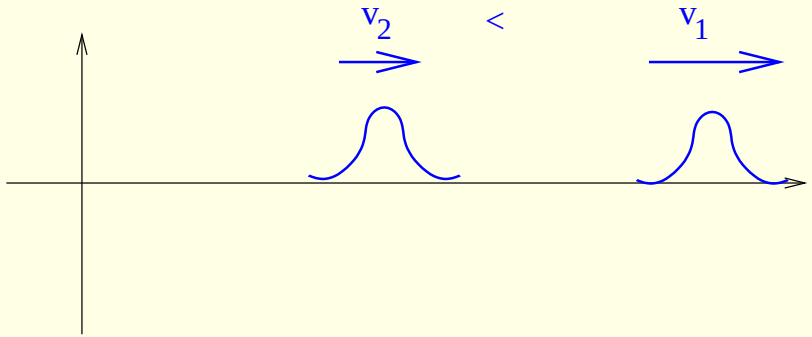

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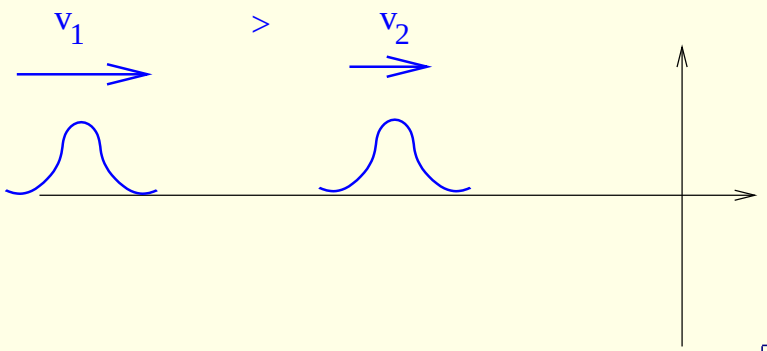
$$|\theta_1, \theta_2\rangle^{in} = S(|\theta_1 - \theta_2|)|\theta_1, \theta_2\rangle^{out}$$

$\theta_1 > \theta_2$

Integrable Field Theory scheme

$v_i = \sinh\theta_i$

Bulk two particle in state: $t \to -\infty$



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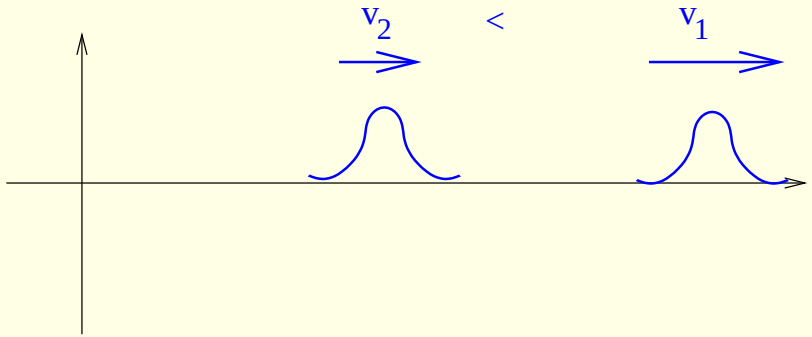

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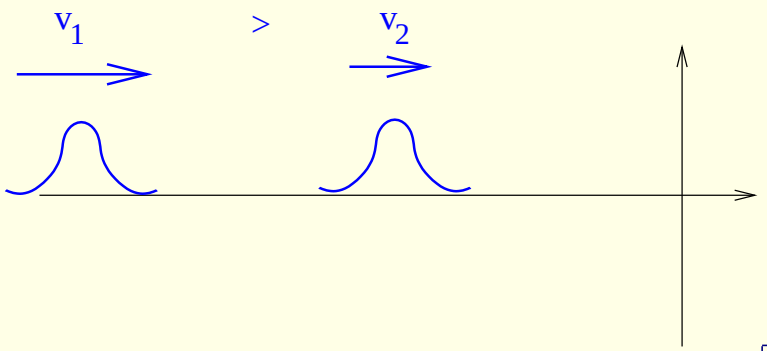

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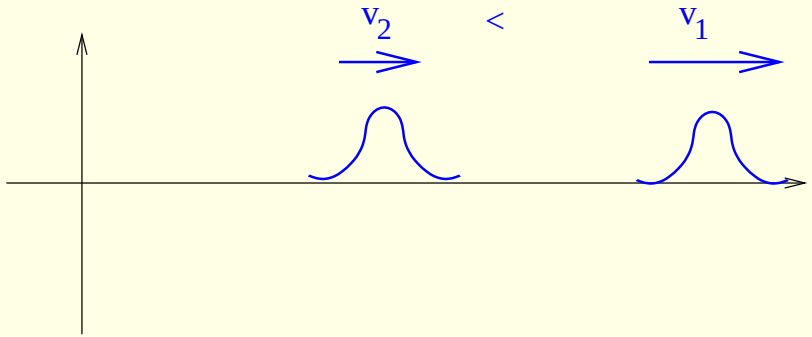

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Integrability $\longrightarrow$ factorizability:

$$|\theta_1,\theta_2,\ldots,\theta_n\rangle = \prod_{i<j} S(\theta_i-\theta_j)|\theta_n,\theta_{n-1},\ldots,\theta_1\rangle$$

Consequences of integrability

Integrability $\longrightarrow$ infinite conserved charges $\longrightarrow$ shift the trajectories

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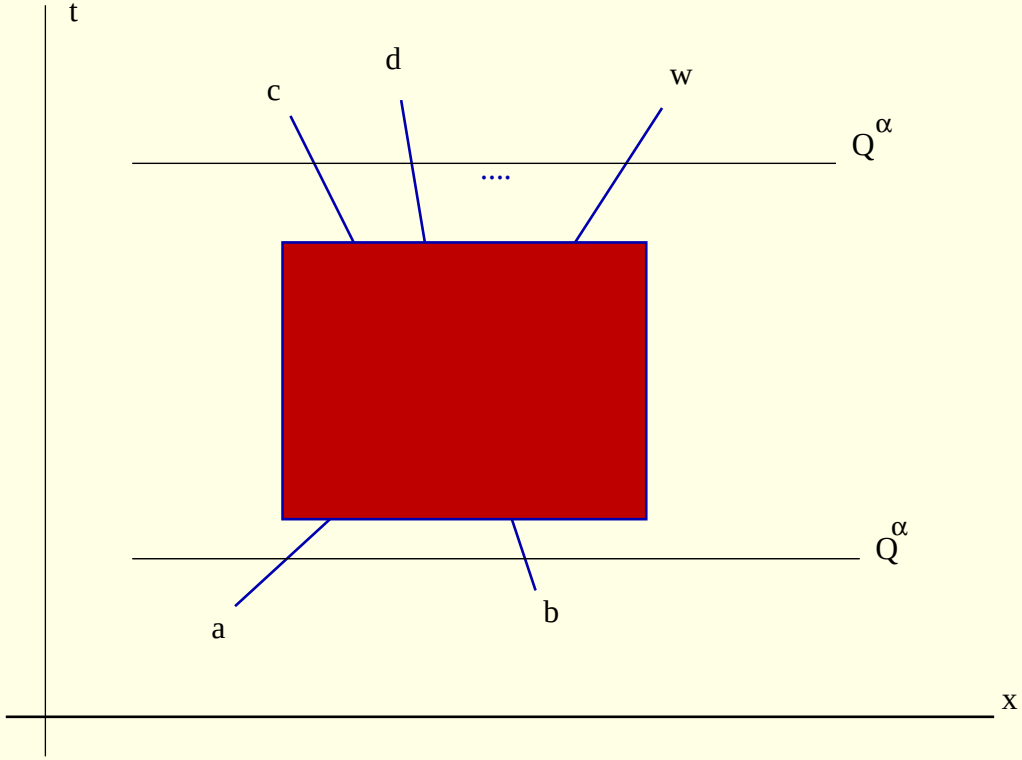

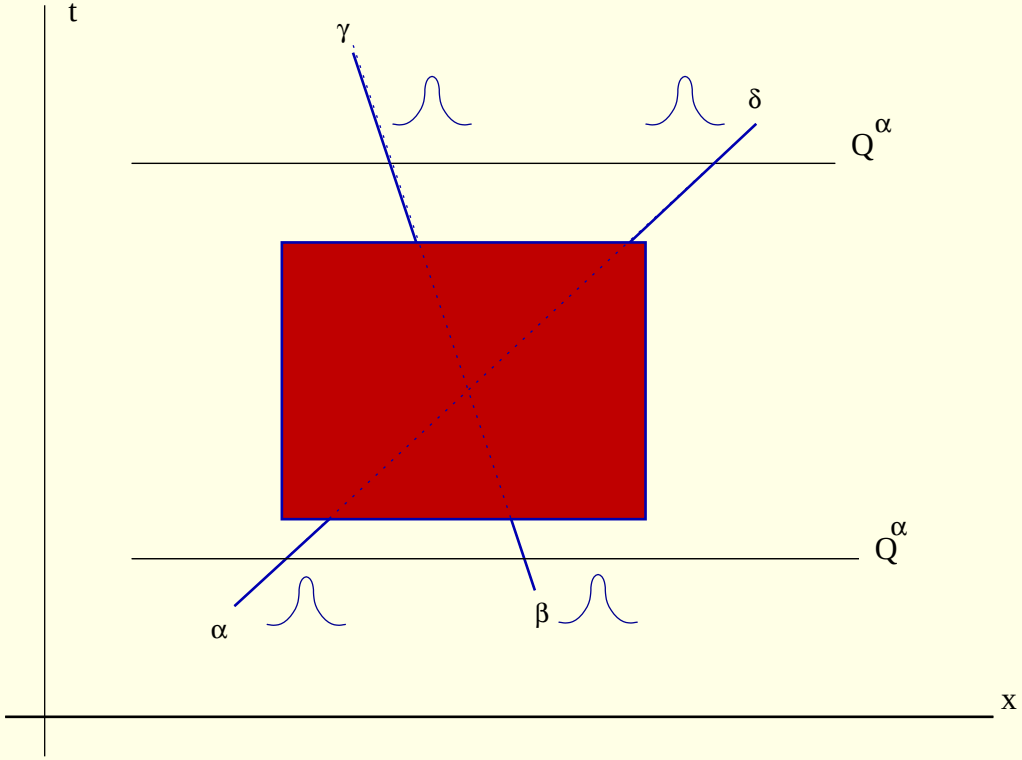

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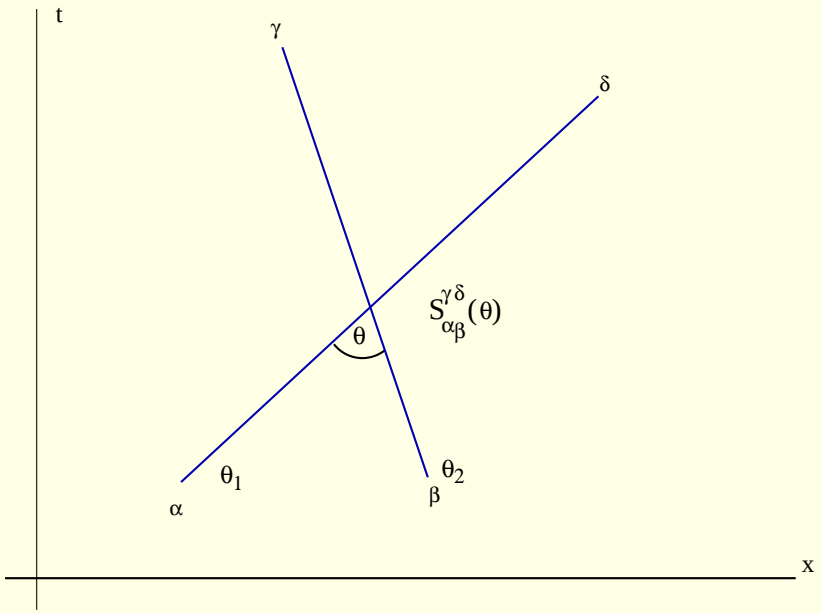

$$S^{\gamma\delta}_{\alpha\beta}(\theta) \qquad \theta = \theta_1 - \theta_2$$

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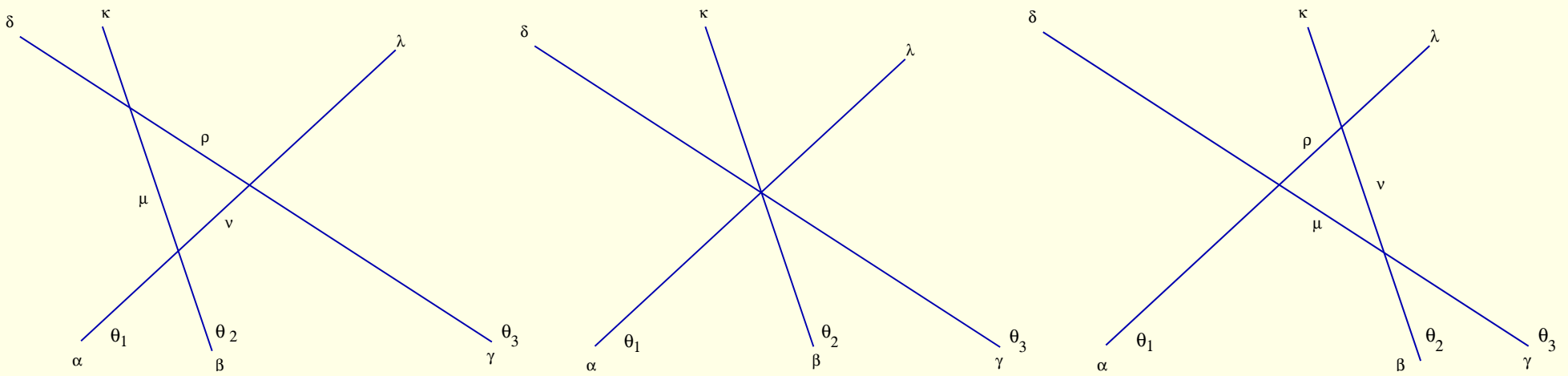

Factorization

$$S^{\delta\kappa\lambda}_{\alpha\beta\gamma}(\theta_1-\theta_2,\theta_2-\theta_3,\theta_1-\theta_3)=S^{\mu\nu}_{\alpha\beta}(\theta_1-\theta_2)S^{\rho\lambda}_{\nu\gamma}(\theta_1-\theta_3)S^{\delta\kappa}_{\mu\rho}(\theta_2-\theta_3)$$

Yang Baxter equation

$$S^{\mu\nu}_{\alpha\beta}(\theta_1-\theta_2)S^{\rho\lambda}_{\nu\gamma}(\theta_1-\theta_3)S^{\delta\kappa}_{\mu\rho}(\theta_2-\theta_3)=S^{\mu\nu}_{\beta\gamma}(\theta_2-\theta_3)S^{\delta\rho}_{\alpha\mu}(\theta_1-\theta_3)S^{\kappa\lambda}_{\rho\nu}(\theta_1-\theta_2)$$

Properties of the S-matrix from the perturbativ scheme

Definition $S(\theta) =_{out} \langle \theta_1, \theta_2 | S | \theta_1, \theta_2 \rangle_{in} = \langle \theta_2, \theta_1 | \theta_1, \theta_2 \rangle$ where $\theta = \theta_1 - \theta_2 > 0$

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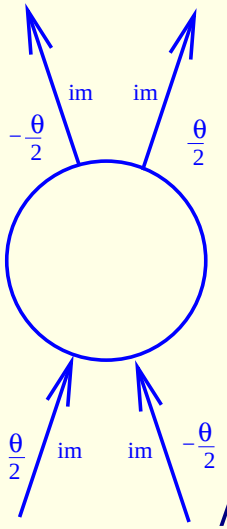

Analytic structure

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(time reversal) $it \to -it$ continuation: $\theta_1 \to i\pi - \theta_1$ Unitarity $S^{-1}(\theta) = S(-\theta)$

Crossing $(x, it) \to (-iT, X)$ $\theta \to \frac{i\pi}{2} - \theta$ Crossing $S(\theta) = S(i\pi - \theta)$

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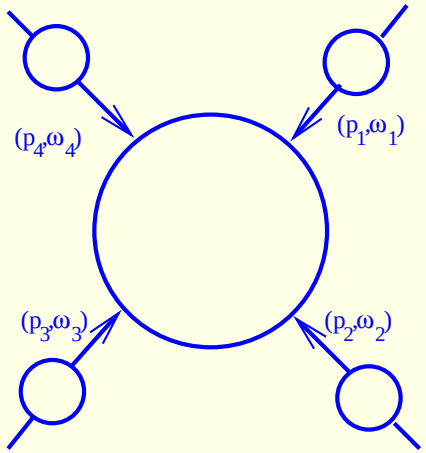
Singularity properties of the correlators: Landau equations

Singularity structure: Landau equations

Singularity structure: Landau equations

Singularity in correlation function$=$ on mass shell particles



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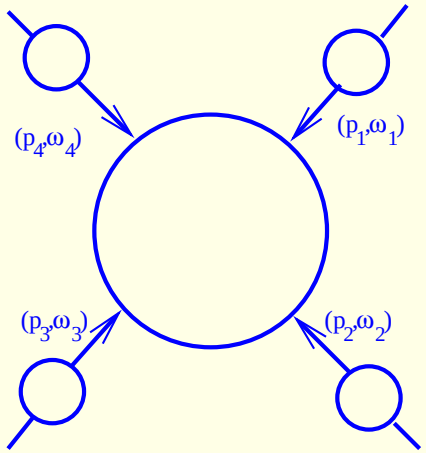

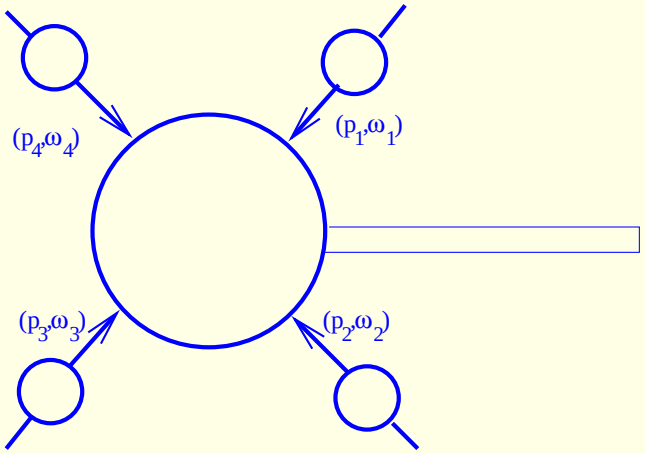

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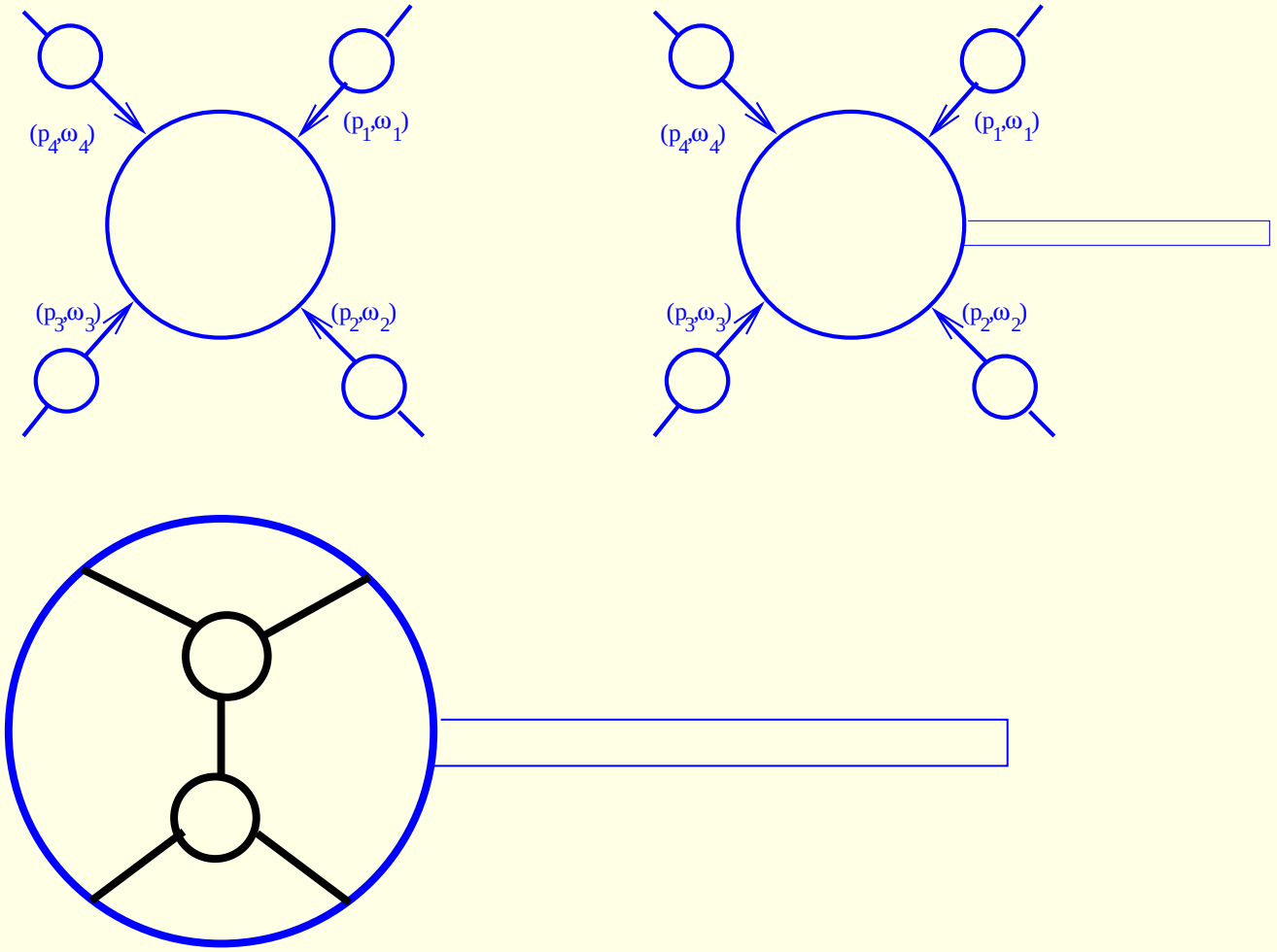

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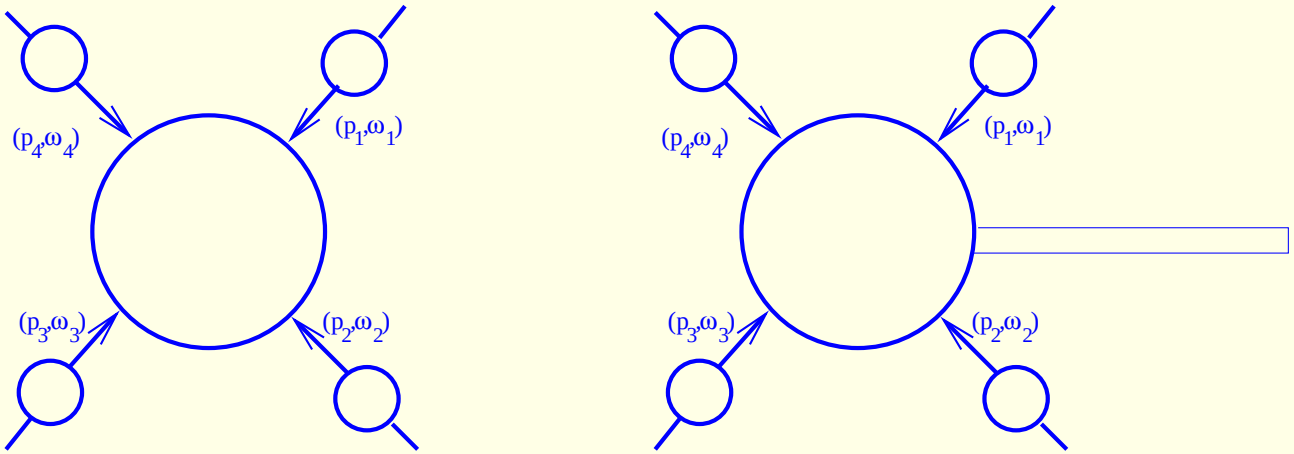
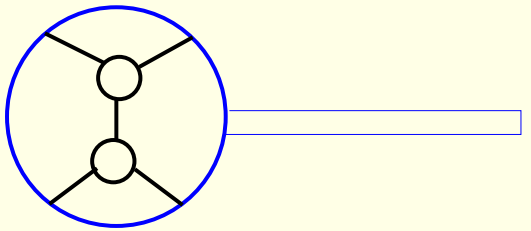


Coleman-Norton interpretation



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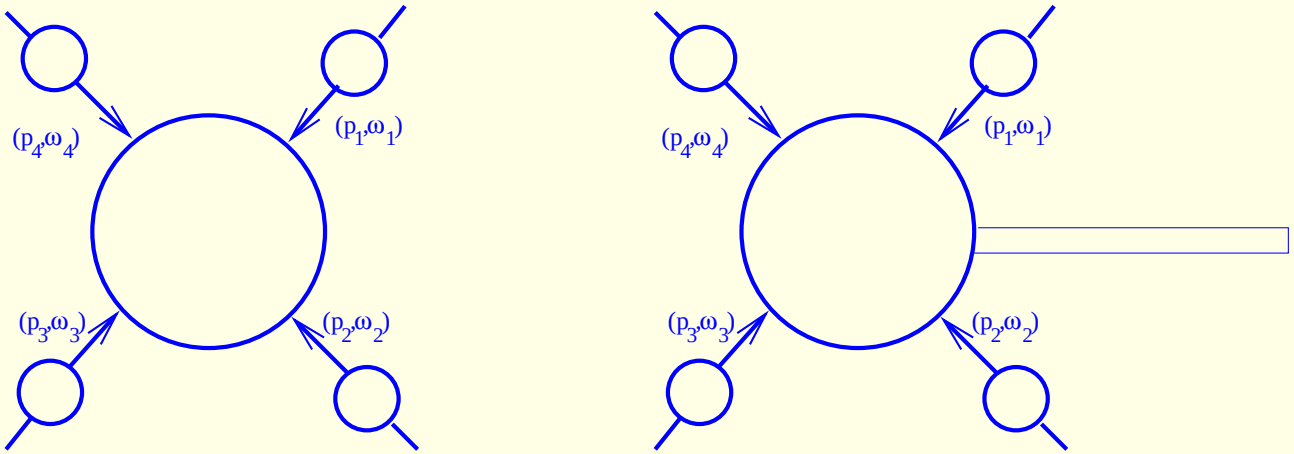

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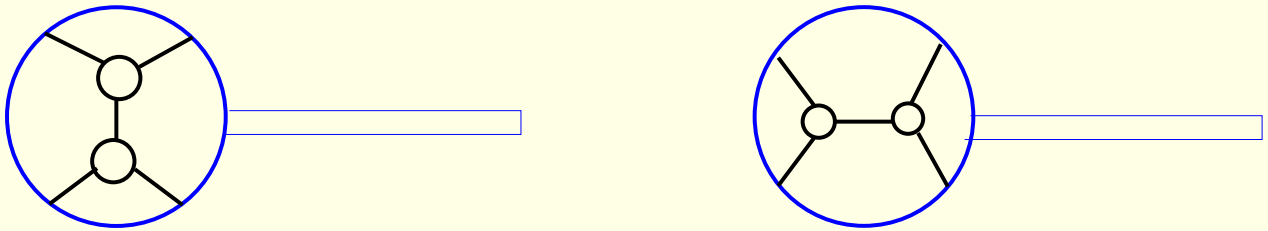

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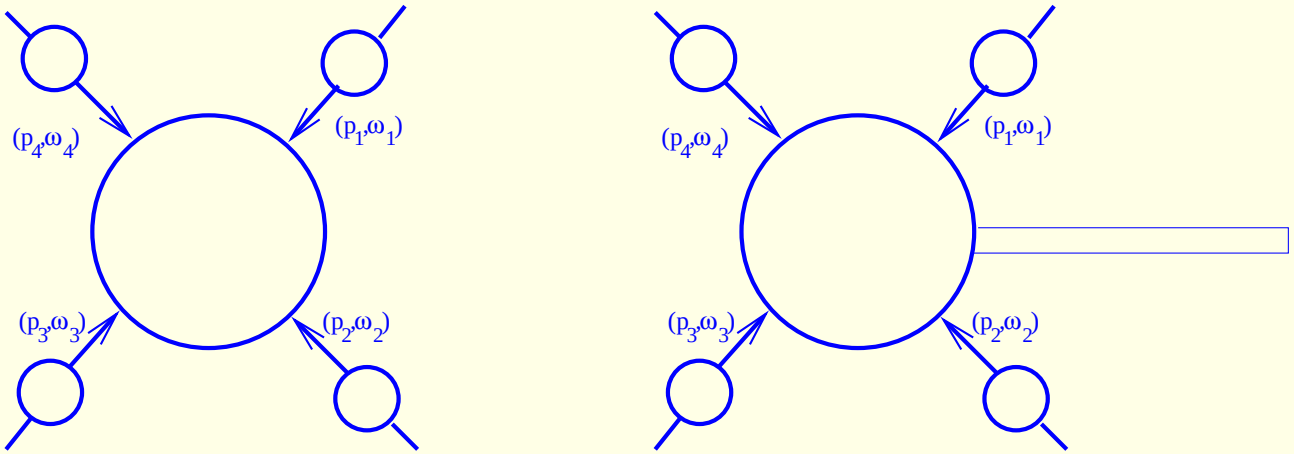

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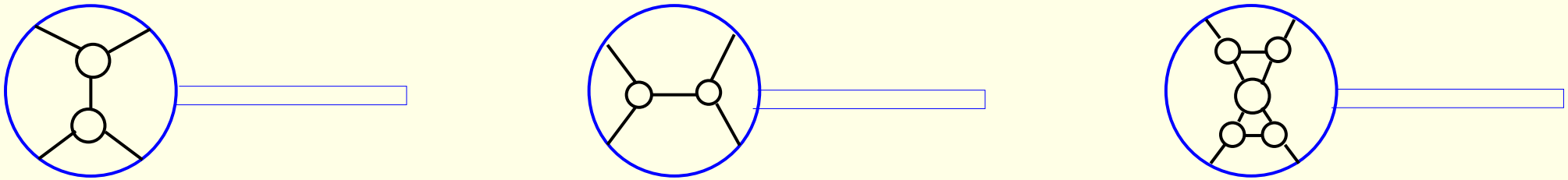

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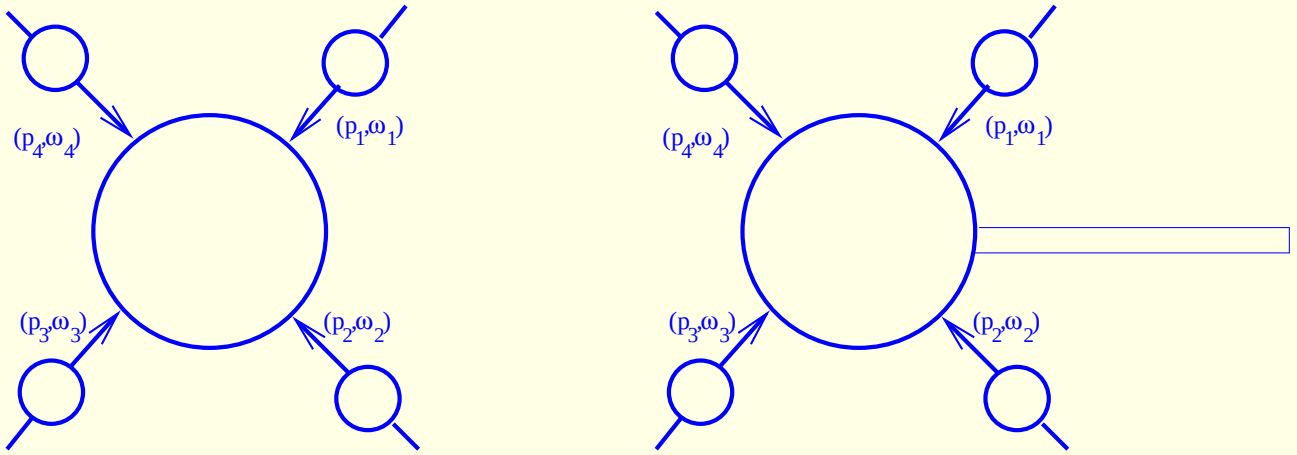

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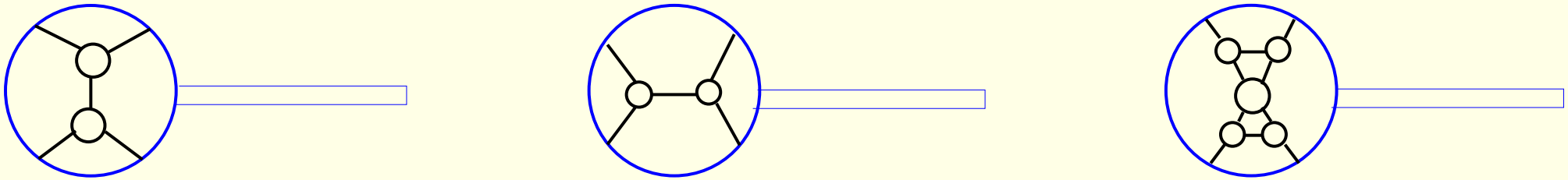

Cutkosky rules: replace $(\omega^2 - p^2 - m^2 + i\epsilon)^{-1}$ with $2\pi\delta^+(\omega^2 - p^2 - m^2)$

$(G^3(u_1))^2$

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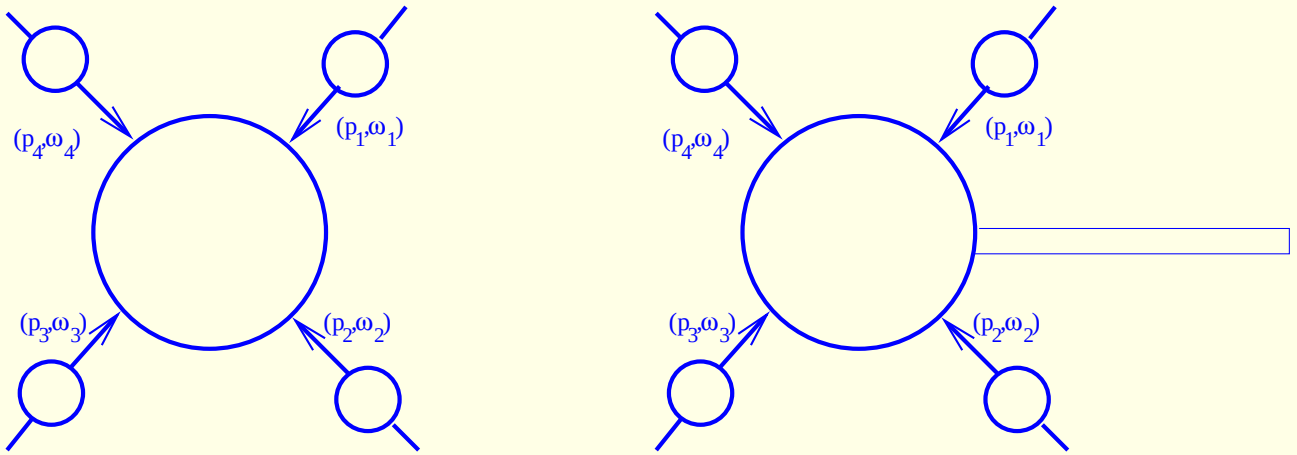

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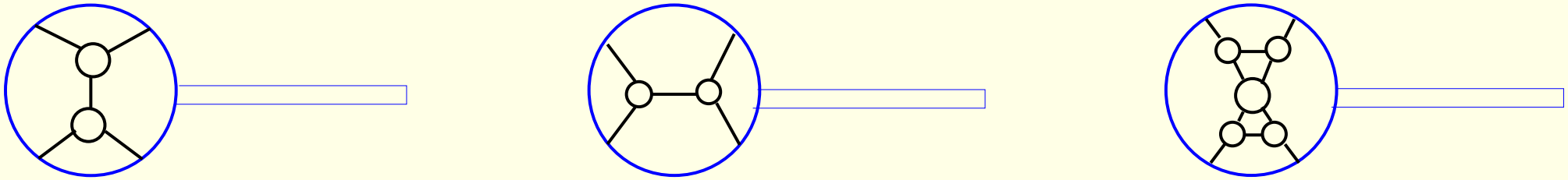

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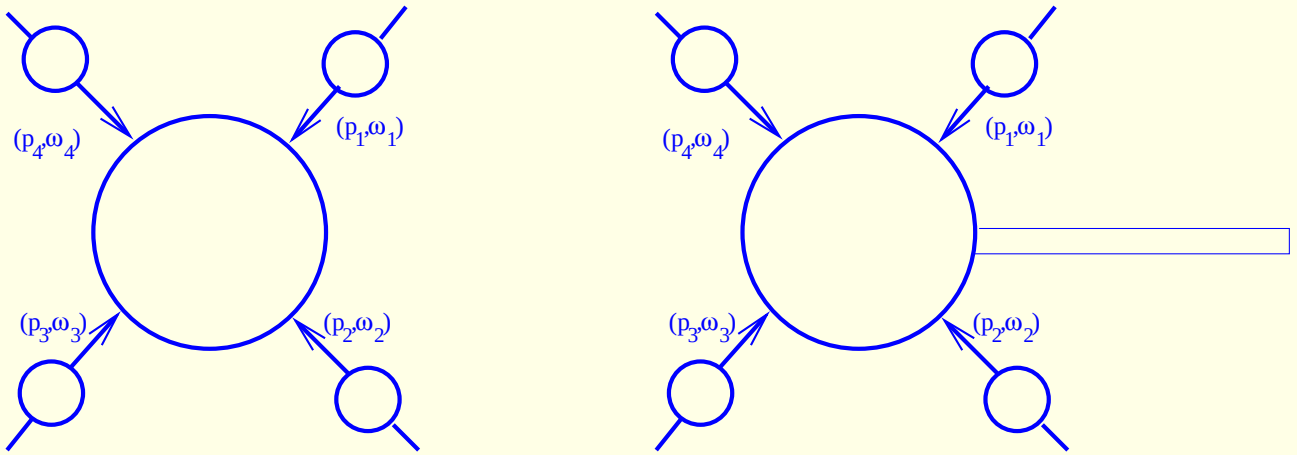

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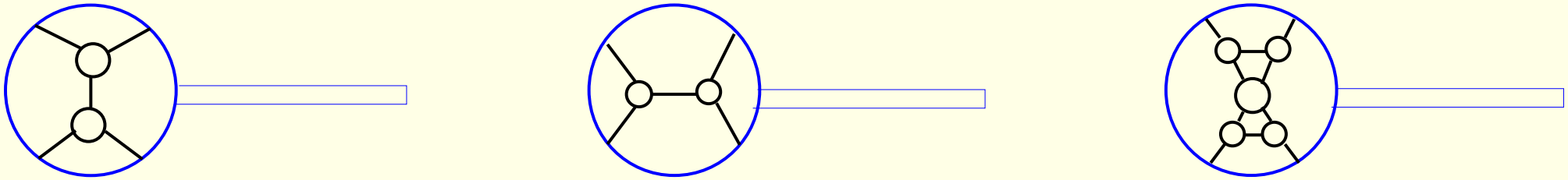

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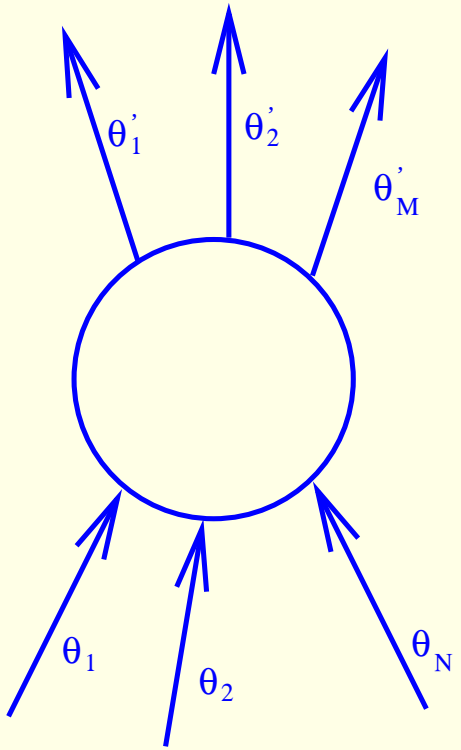
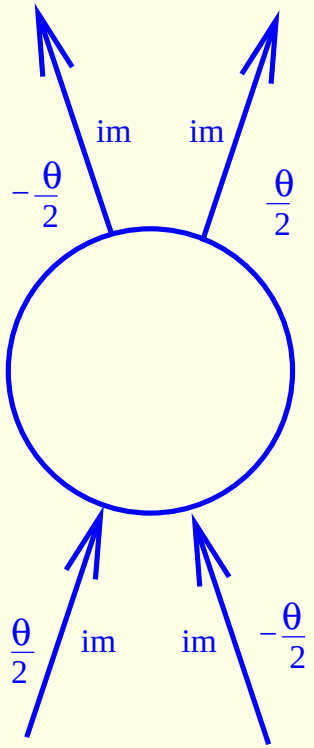
S-matrix bootstrap

S-matrix bootstrap

Integrability: shifting the trajectories $\longrightarrow$factorization + bootstrap

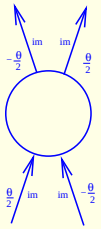
The only nontrivial scattering matrix $S_2^2(|\theta_1 - \theta_2|) = S(\theta)$ $\quad \boxed{\theta > 0}$

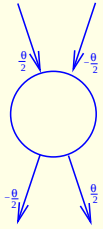




S-matrix bootstrap

Unitarity

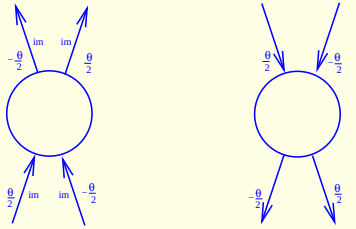




$$S(\theta)^{-1} = S(-\theta)$$

S-matrix bootstrap

Unitarity



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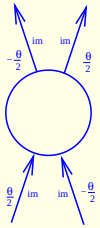
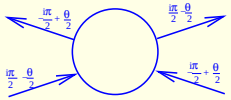
Crossing





$$S(\theta) = S(i\pi - \theta)$$

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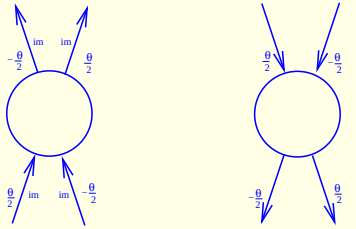

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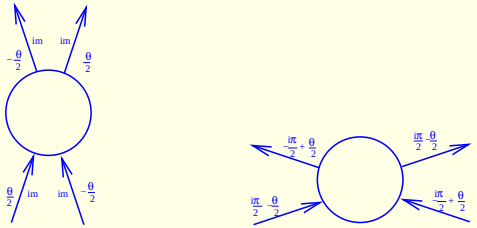

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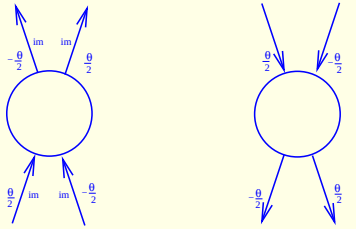
Simplest nontrivial solution $S(\theta) = \frac{\sinh\theta - i\sin\gamma}{\sinh\theta + i\sin\gamma}$

S-matrix bootstrap

Unitarity



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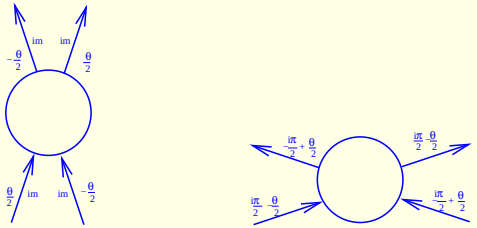

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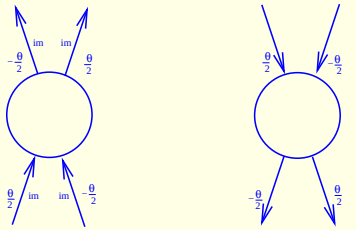
Simplest nontrivial solution $S(\theta) = \frac{\sinh\theta - i\sin\gamma}{\sinh\theta + i\sin\gamma}$

Minimality: all singularity has physical origin: $\gamma > 0$ end of the story (sinh-Gordon):

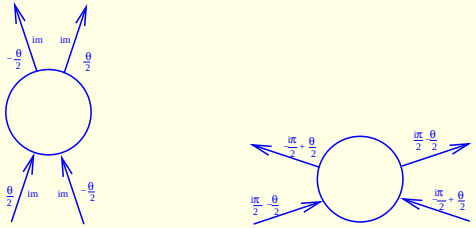
S-matrix bootstrap

Unitarity



$$S(\theta)^{-1} = S(-\theta)$$

Crossing



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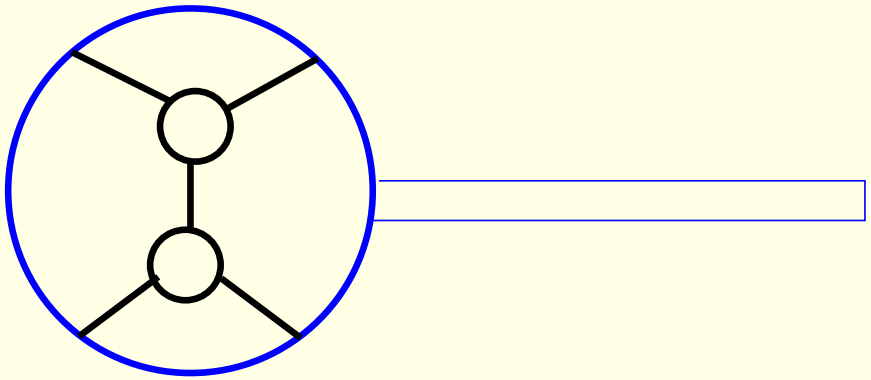

$\gamma < 0$ boundstate

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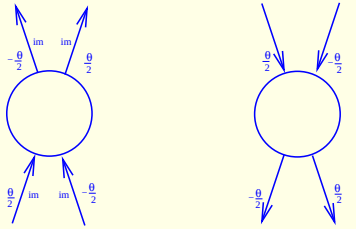
Unitarity



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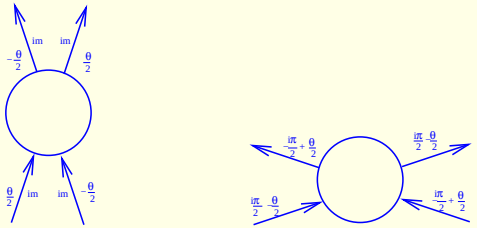

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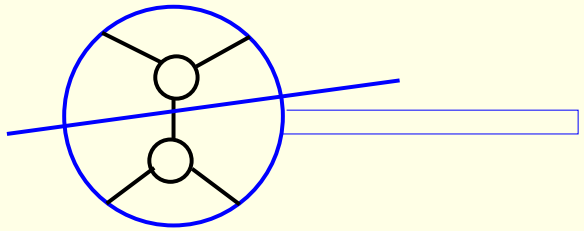

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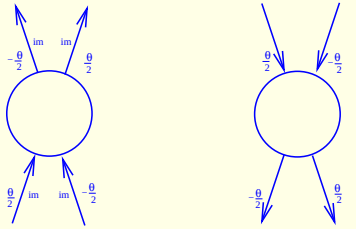

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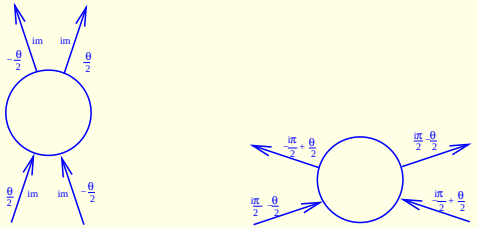

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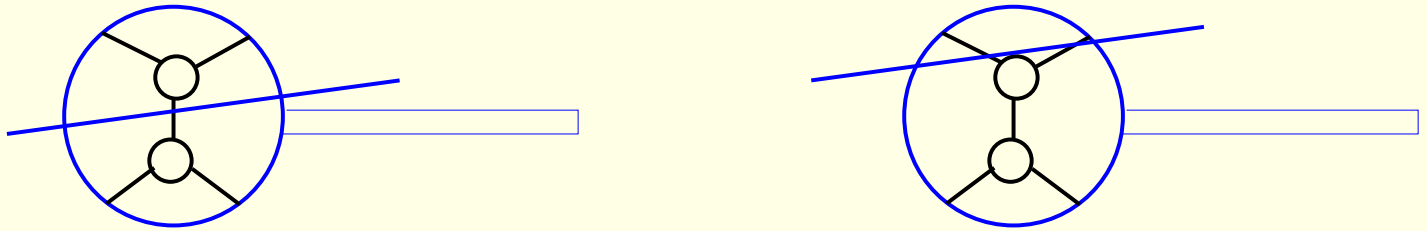

S-matrix bootstrap

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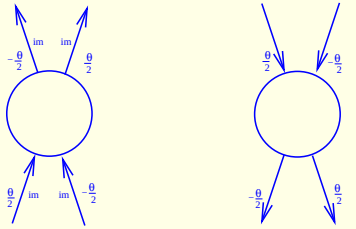

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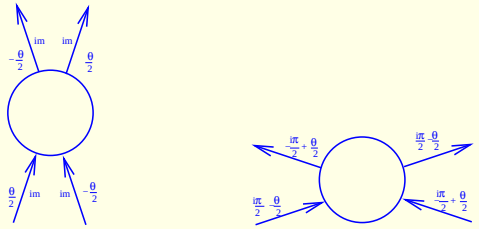

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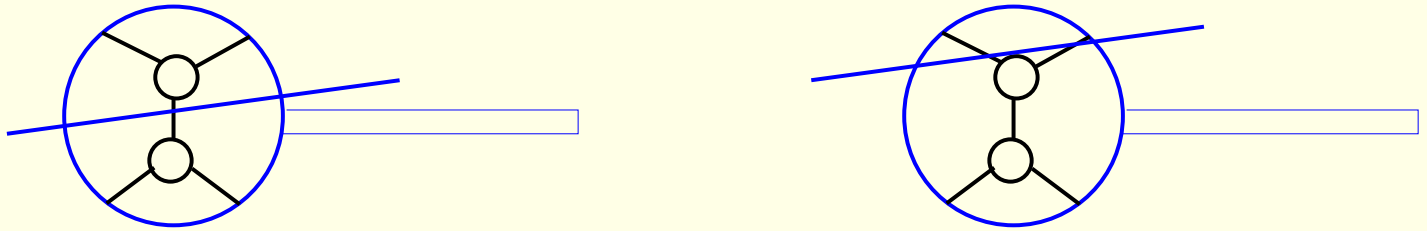
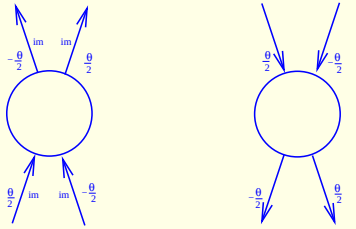


$S_{new}(\theta)$

S-matrix bootstrap

Unitarity



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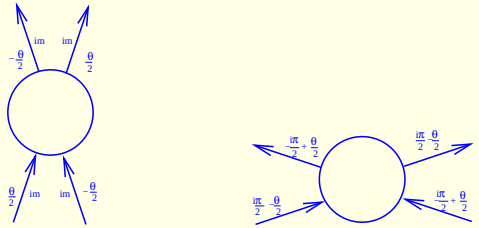

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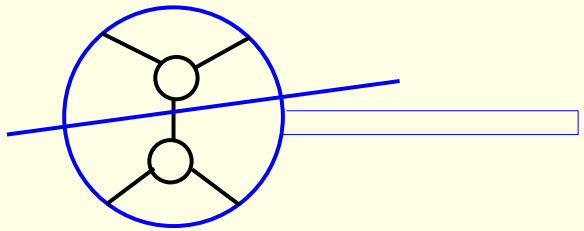
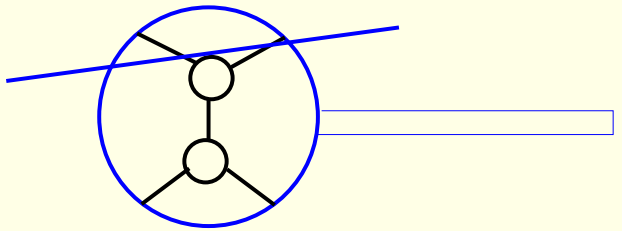


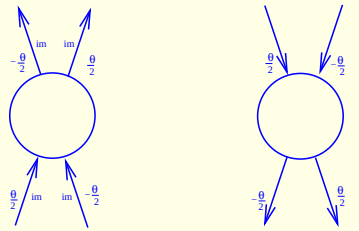
$S_{new}(\theta)$



$S_{old}(\theta + iu)S_{old}(\theta - iu)$

S-matrix bootstrap

Unitarity



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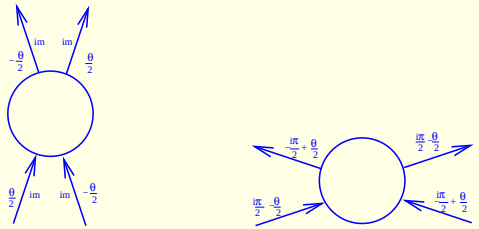

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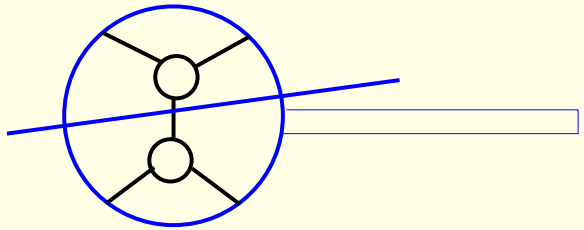
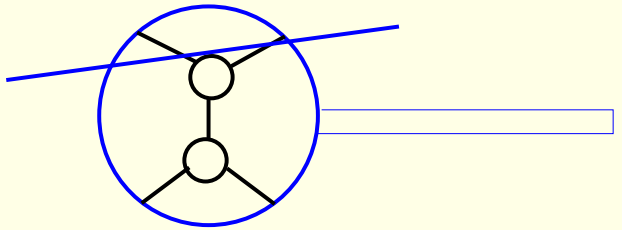


$S_{new}(\theta)$



$S_{old}(\theta + iu)S_{old}(\theta - iu)$

for $\gamma = -\frac{2}{3}$ self-fusion: Lee-Yang,
for generic γ sine-Gordon $B_2, B_3, \ldots, B_n, s, \bar{s}$