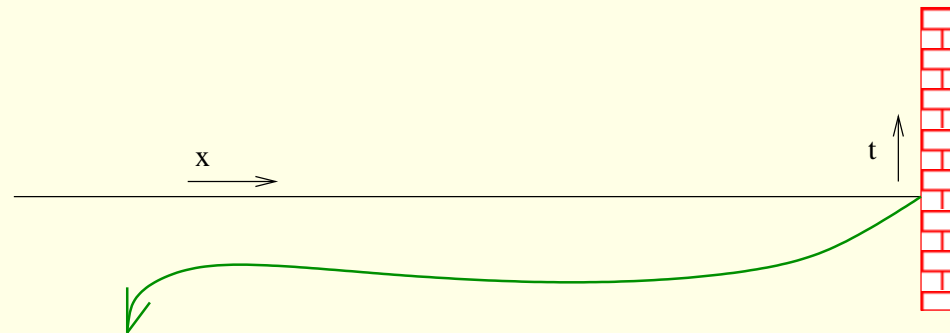


Bulk and Boundary Form Factors in QFT (3)

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Massive integrable **boundary** QFT in 1+1 D (diagonal)



$${}_B \langle 0 | \varphi_B(t) | \theta_1, \theta_2, \dots, \theta_n \rangle_B^{in}$$

Plan of talk

$${}_B \langle 0 | \varphi_B(t) | \theta_1, \theta_2, \dots, \theta_n \rangle_B^{in}$$

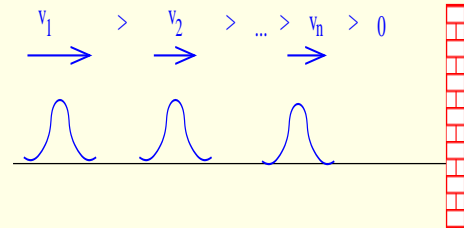
Boundary form factor

$\varphi_B(t)$: local boundary operator

Bulk $\langle \rangle$
Bulk form factors + $|B\rangle$

Boundary $\langle \rangle$
Boundary form factors!

$|\theta_1, \theta_2, \dots, \theta_n\rangle_B^{in}$
Asymptotic states \rangle



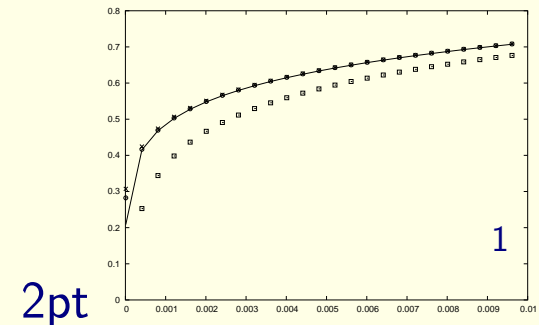
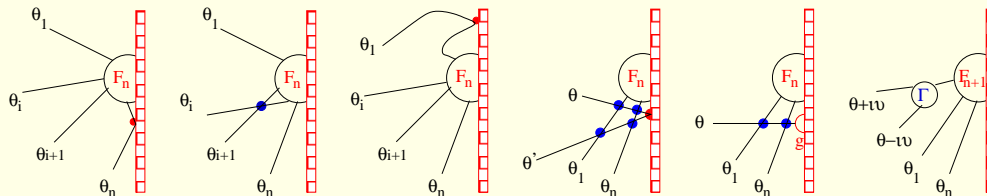
Boundary reduction formula \rangle

Consistency equations !

\rightarrow Axioms \rangle

Minimal solution \rangle

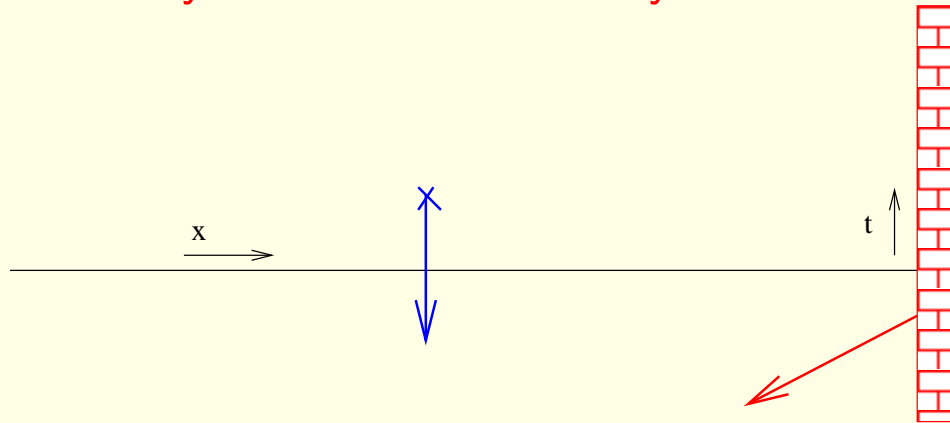
Perturbed boundary Lee-Yang



2pt

Integrable massive BQFT as perturbed BCFT

Boundary conformal field theory



Operator - state correspondence

$$\{\varphi_B(t)\} \leftrightarrow \mathcal{H}_B$$

Bulk operators $\Phi(x, t)$

$\varphi_B(t)$ Boundary operators

Bulk - Boundary OPE

Integrable perturbations

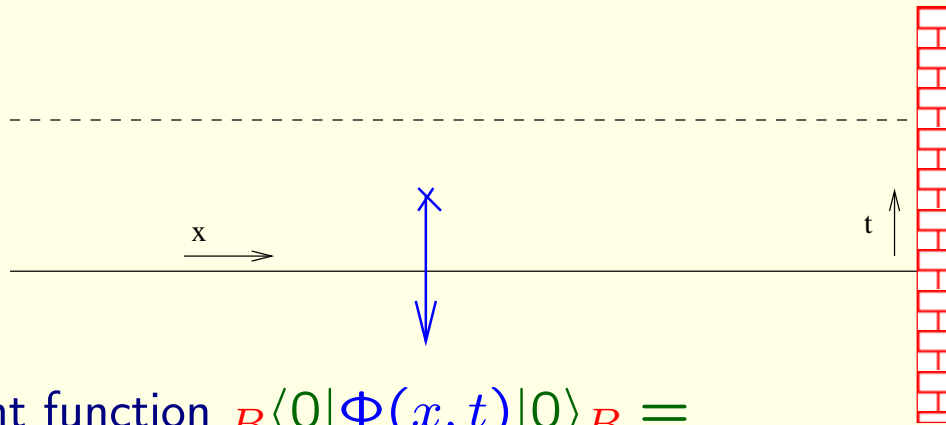
$$S = S_{BCFT} - \lambda_{bulk} \int dt \int_{-\infty}^0 dx \Phi(x, t) - \lambda_{bdry} \int dt \varphi_B(t)$$

Hilbert space does not change

Assumptions: spectrum, operator algebra smoothly changes \rightarrow

Operator content
 Bulk operators $\Phi(x, t)$
 Boundary operators $\varphi_B(t)$

Correlation functions I: bulk operators



One point function ${}_B\langle 0 | \Phi(x, t) | 0 \rangle_B =$

$$\sum_{n=0}^{\infty} \int_{-\infty}^{\infty} \frac{d\theta_1 \dots d\theta_n}{n! (2\pi)^n} \langle 0 | \Phi(x, t) | \theta_1, \dots, \theta_n \rangle \langle \theta_1, \dots, \theta_n | B \rangle e^{-m x \sum_{i=1}^n \cosh \theta_i}$$



Bulk form factor

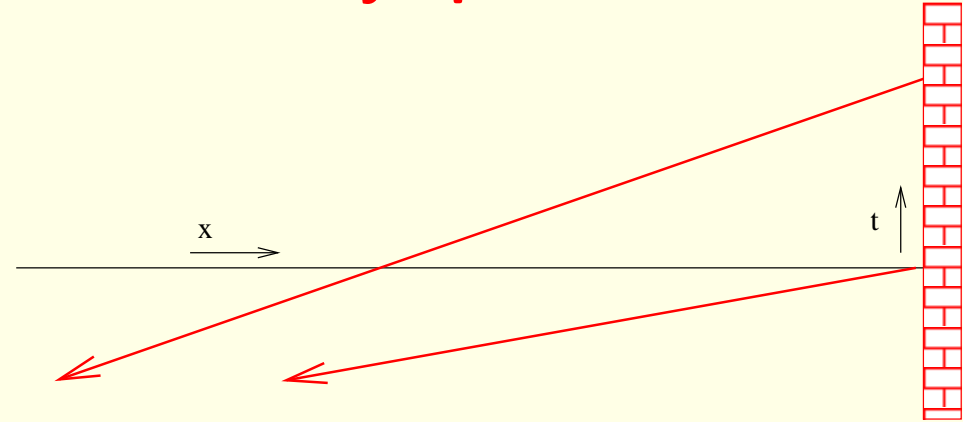


boundary state

Expansion for large x using bulk form factors and the boundary state

Boundary operators $x = 0$ \rightarrow new (boundary) technic is needed

Correlation functions II: boundary operators



Boundary two point function

$${}_B \langle 0 | \varphi_B(t) \varphi_B(0) | 0 \rangle_B =$$

$$\sum_{n=0}^{\infty} \int_0^{\infty} \frac{d\theta_1 \dots d\theta_n}{n! (2\pi)^n} {}_B \langle 0 | \varphi_B(t) | \theta_1, \dots, \theta_n \rangle_B {}_B \langle \theta_1, \dots, \theta_n | \varphi_B(0) | 0 \rangle_B$$

time dependence:

$$\sum_{n=0}^{\infty} \int_0^{\infty} \frac{d\theta_1 \dots d\theta_n}{n! (2\pi)^n} |F_n^{\varphi_B}(\theta_1, \dots, \theta_n)|^2 e^{-mt} \sum_{i=1}^n \cosh \theta_i$$

boundary form factor

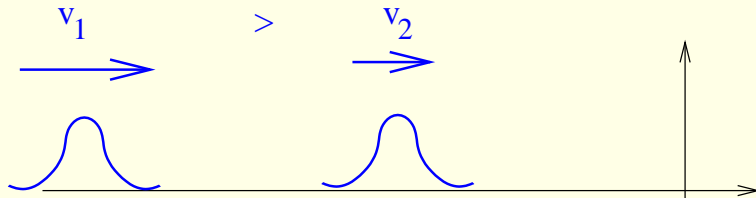
$$F_n^{\varphi_B}(\theta_1, \dots, \theta_n) = {}_B \langle 0 | \varphi_B(t) | \theta_1, \dots, \theta_n \rangle_B^{in}$$

Boundary correlation functions, large t expansion \rightarrow boundary form factors $| \rangle$

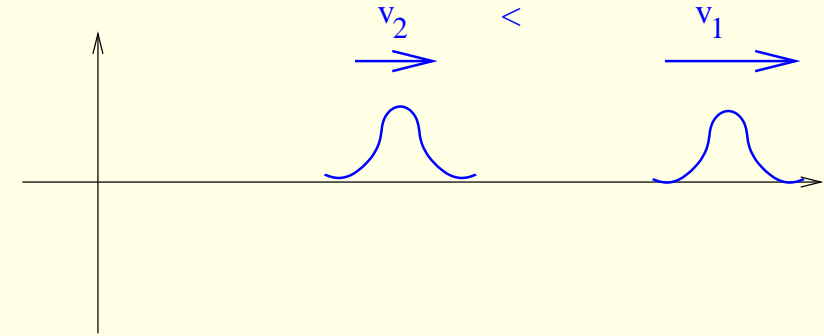
Asymptotic states: $|\theta_1, \theta_2, \dots, \theta_n\rangle^{in}$

$$v_i = \sinh \theta_i$$

Bulk two particle in state: $t \rightarrow -\infty$



Bulk two particle out state: $t \rightarrow \infty$



Free, noninteracting in particles

S-matrix

Free, noninteracting out particles

$$|\theta_1, \theta_2\rangle^{in}$$

=

$$S(|\theta_1 - \theta_2|)|\theta_1, \theta_2\rangle^{out}$$

$$\theta_1 > \theta_2$$

$$|\theta_1, \theta_2\rangle$$

=

$$S(\theta_1 - \theta_2)|\theta_2, \theta_1\rangle$$

Unitarity $S^*(\theta_1 - \theta_2) = S^{-1}(\theta_1 - \theta_2) = S(\theta_2 - \theta_1)$

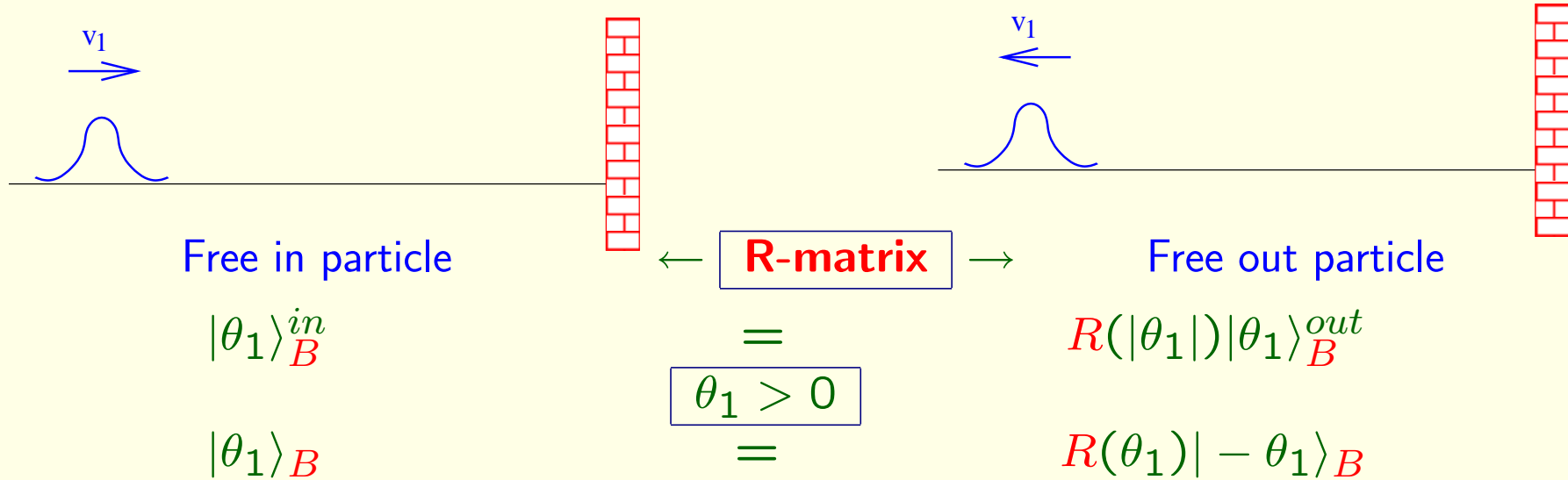
Integrability \rightarrow factorizability:

$$|\theta_1, \theta_2, \dots, \theta_n\rangle = \prod_{i < j} S(\theta_i - \theta_j) |\theta_n, \theta_{n-1}, \dots, \theta_1\rangle$$

Boundary asymptotic states: $|\theta_1, \theta_2, \dots, \theta_n\rangle_B^{in}$

Boundary one particle in state: $t \rightarrow -\infty$

Boundary one pt out state: $t \rightarrow \infty$



Unitarity: $R^*(\theta) = R^{-1}(\theta) = R(-\theta)$

Integrability \rightarrow factorizability:

$$|\theta_1, \theta_2, \dots, \theta_n\rangle_B = \prod_{i < j} S(\theta_i - \theta_j) \prod_i R(\theta_i) | -\theta_1, -\theta_2, \dots, -\theta_n\rangle_B$$

!>

Analytic structure: reduction formula

Unitarity $S^*(\theta_1 - \theta_2) = S^{-1}(\theta_1 - \theta_2) = S(\theta_2 - \theta_1)$ not restrictive enough



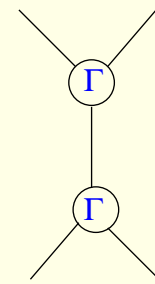
$$\langle \theta_1, \theta_2, \dots, \theta_n | \mathcal{O}(x, t) | in \rangle = \delta(\theta_1 - \theta) \langle \theta_2, \dots, \theta_n | \mathcal{O}(x, t) | in \setminus \theta \rangle$$

$$iZ^{-1/2} \int_{-\infty}^{\infty} dt' e^{-i\omega(\theta_1)t'} \int_{-\infty}^{\infty} dx' e^{ip(\theta_1)x'} \{ \partial_{t'}^2 - \partial_{x'}^2 + m^2 \}$$

$$\langle \theta_2, \dots, \theta_n | T(\Phi(x', t') \mathcal{O}(x, t)) | in \rangle$$

Crossing $S(\theta_1 - \theta_2) = S(i\pi - \theta_1 + \theta_2)$

perturbed Lee Yang model $S_{LY}(\theta) = \frac{\sinh \theta + i \sin \frac{\pi}{3}}{\sinh \theta - i \sin \frac{\pi}{3}}$



Analytic structure: boundary reduction formula

Unitarity $R^*(\theta_1) = R^{-1}(\theta_1) = R(-\theta_1)$ not restrictive enough



$$B < \theta_1, \theta_2, \dots, \theta_n | \mathcal{O}(t) | in >_B = \delta(\theta_1 - \theta) B < \theta_2, \dots, \theta_n | \mathcal{O}(t) | in \setminus \theta >_B$$

$$i2Z^{-1/2} \int_{-\infty}^{\infty} dt' e^{-i\omega(\theta_1)t'} \int_{-\infty}^0 dx' \cos(p(\theta_1)x') \{ \partial_{t'}^2 - \partial_x^2 + m^2 + \delta(x') \partial_{x'} \}$$

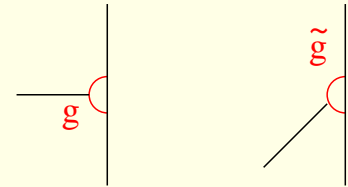
$$B < \theta_2, \dots, \theta_n | T(\Phi(x', t') \mathcal{O}(t)) | in >_B$$

Boundary crossing $R(\theta_1) = S(2\theta_1)R(i\pi - \theta_1)$

perturbed boundary Lee Yang

$$S(\theta) = - \begin{pmatrix} 1 \\ 3 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix} = - \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \quad [x] = (x)(1-x) \quad ; \quad (x) = \frac{\sinh(\frac{\theta}{2} + \frac{i\pi x}{2})}{\sinh(\frac{\theta}{2} - \frac{i\pi x}{2})}$$

$$R(\theta) = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \begin{pmatrix} 1 \\ 6 \end{pmatrix} \begin{pmatrix} -2 \\ 3 \end{pmatrix} \begin{bmatrix} b+1 \\ 6 \end{bmatrix} \begin{bmatrix} b-1 \\ 6 \end{bmatrix}$$



Analytical properties of the bulk form factors

Bulk form factor

$$\theta_1 > \theta_2 > \dots > \theta_n$$

$$F_n^\circ(\theta_1, \dots, \theta_n) = \langle 0 | \mathcal{O}(0, 0) | \theta_1, \dots, \theta_n \rangle$$

$$F_n^\circ(\theta_n, \dots, \theta_1) = \langle 0 | \mathcal{O}(0, 0) | \theta_n, \dots, \theta_1 \rangle$$

Example

$$F_2^\circ(\theta_1, \theta_2) = S(\theta_1 - \theta_2) F_2^\circ(\theta_2, \theta_1)$$

$$F_{mn}^\circ(\theta'_1, \dots, \theta'_m | \theta_1, \dots, \theta_n) = \langle \theta'_1, \dots, \theta'_m | \mathcal{O}(0, 0) | \theta_1, \dots, \theta_n \rangle$$

Permutation

$$F_{m2}^\circ(\theta'_1, \dots, \theta'_m | \theta_1, \theta_2) = S(\theta_1 - \theta_2) F_{m2}^\circ(\theta'_1, \dots, \theta'_m | \theta_2, \theta_1)$$

Crossing from reduction formula

$$F_{1n}^\circ(\theta | \theta_1, \dots, \theta_n) = F_{n+1}^\circ(\theta_1, \dots, \theta_n, \theta - i\pi) + \delta(\theta - \theta_n) F_{n-1}^\circ(\theta_1, \dots, \theta_{n-1})$$

$$F_{1n}^\circ(\theta | \theta_n, \dots, \theta_1) = F_{n+1}^\circ(\theta + i\pi, \theta_n, \dots, \theta_1) + \delta(\theta - \theta_n) F_{n-1}^\circ(\theta_{n-1}, \dots, \theta_1)$$

!>

Analytical properties of the boundary form factors

Boundary form factor

$$\theta_1 > \theta_2 > \dots > \theta_n > 0$$

$$F_n^{\mathcal{O}}(\theta_1, \dots, \theta_n) =_B \langle 0 | \mathcal{O}(0) | \theta_1, \dots, \theta_n \rangle_B$$

$$F_n^{\mathcal{O}}(-\theta_1, \dots, -\theta_n) =_B \langle 0 | \mathcal{O}(0) | -\theta_1, \dots, -\theta_n \rangle_B$$

Example

$$F_1^{\mathcal{O}}(\theta_1) = R(\theta_1) F_1^{\mathcal{O}}(-\theta_1)$$

$$F_{mn}^{\mathcal{O}}(\theta'_1, \dots, \theta'_m | \theta_1, \dots, \theta_n) =_B \langle \theta'_1, \dots, \theta'_m | \mathcal{O}(0) | \theta_1, \dots, \theta_n \rangle_B$$

Reflection

$$F_{m1}^{\mathcal{O}}(\theta'_1, \dots, \theta'_m | \theta_1) = R(\theta_1) F_{m1}^{\mathcal{O}}(\theta'_1, \dots, \theta'_m | -\theta_1) + \frac{g}{2} \delta(\theta - \frac{i\pi}{2}) F_{m0}^{\mathcal{O}}(\theta'_1, \dots, \theta'_m)$$

$$F_{1n}^{\mathcal{O}}(\theta'_1 | \theta_1, \dots, \theta_n) = R(-\theta'_1) F_{1n}^{\mathcal{O}}(-\theta'_1 | \theta_1, \dots, \theta_n) + \frac{g}{2} \delta(\theta' + \frac{i\pi}{2}) F_{0n}^{\mathcal{O}}(\theta_1, \dots, \theta_n)$$

Crossing from
reduction formula

$$F_{1n}^{\mathcal{O}}(\theta' | \theta_1, \dots, \theta_n) = F_{n+1}^{\mathcal{O}}(\theta' + i\pi, \theta_1, \dots, \theta_n) + \delta(\theta - \theta_1) F_{n-1}^{\mathcal{O}}(\theta_2, \dots, \theta_n)$$

$$F_{1n}^{\mathcal{O}}(-\theta' | -\theta_1, \dots, -\theta_n) = F_{n+1}^{\mathcal{O}}(-\theta' + i\pi, -\theta_1, \dots, -\theta_n) + \delta(\theta' - \theta_1) F_{n-1}^{\mathcal{O}}(-\theta_2, \dots, -\theta_n)$$

Comparison to other approaches

Consistency eqs.:
derived

$$F_{m1}^{\circ}(\theta'_1, \dots, \theta'_m | \theta_1) = R(\theta_1) F_{m1}^{\circ}(\theta'_1, \dots, \theta'_m | -\theta_1) + \frac{g}{2} \delta(\theta - \frac{i\pi}{2}) F_{m0}^{\circ}(\theta'_1, \dots, \theta'_m)$$

$$F_{1n}^{\circ}(\theta'_1 | \theta_1, \dots, \theta_n) = R(-\theta'_1) F_{1n}^{\circ}(-\theta'_1 | \theta_1, \dots, \theta_n) + \frac{g}{2} \delta(\theta' + \frac{i\pi}{2}) F_{0n}^{\circ}(\theta_1, \dots, \theta_n)$$

$$F_{1n}^{\circ}(\theta' | \theta_1, \dots, \theta_n) = F_{n+1}^{\circ}(\theta' + i\pi, \theta_1, \dots, \theta_n) + \delta(\theta - \theta_1) F_{n-1}^{\circ}(\theta_2, \dots, \theta_n)$$

$$F_{1n}^{\circ}(-\theta' | -\theta_1, \dots, -\theta_n) = F_{n+1}^{\circ}(-\theta' + i\pi, -\theta_1, \dots, -\theta_n) + \delta(\theta' - \theta_1) F_{n-1}^{\circ}(-\theta_2, \dots, -\theta_n)$$

Similar equations in spin models **BUT WITHOUT** $\frac{g}{2} \delta(\theta - \frac{i\pi}{2}) \dots$:

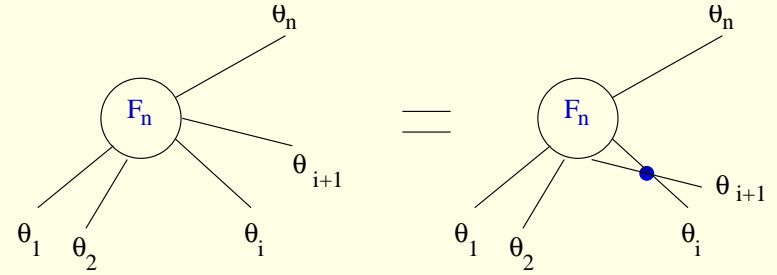
XXZ, XYZ	M. Jimbo, R. Kedem, H. Konno T. Miwa, R. Weston	Nucl.Phys. B448 (1995) 429-456
Higher rank XXZ	Y.-H. Quano	Int.J.Mod.Phys. A15 (2000) 3699-3716
Belavin's Z_n -symmetric	Y.-H. Quano	J.Phys. A33 (2000) 8275
$A_{n-1}^{(1)}$ face model	Y.-H. Quano	J.Phys. A34 (2001) 8445-8464

Equations are DERIVED \rightarrow valid for any QFT for form factors of local operators!

Bulk form factor axioms

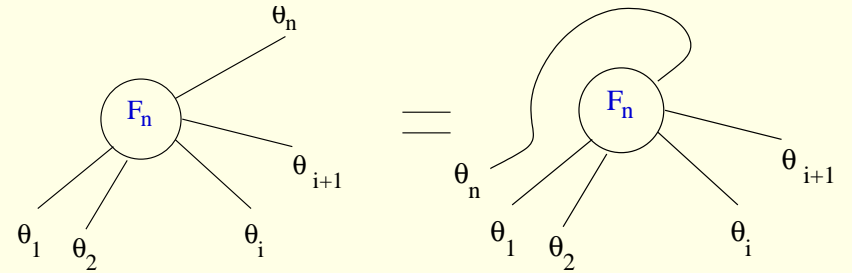
Permutation

$$F_n^\circ(\theta_1, \dots, \theta_i, \theta_{i+1}, \dots, \theta_n) = S(\theta_i - \theta_{i+1}) F_n^\circ(\theta_1, \dots, \theta_{i+1}, \theta_i, \dots, \theta_n)$$



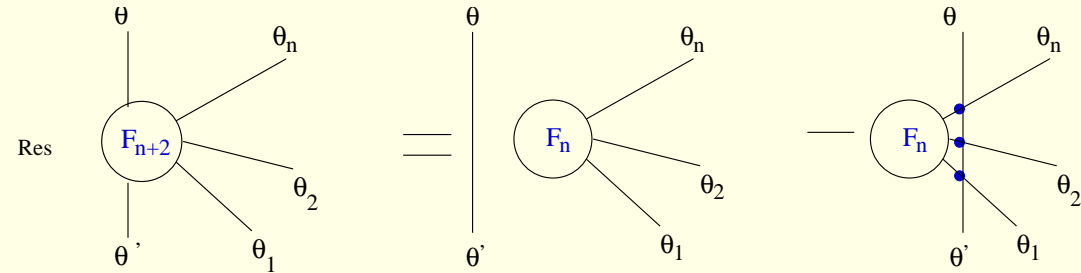
Periodicity

$$F_n^\circ(\theta_1, \dots, \theta_{n-1}, \theta_n) = F_n^\circ(\theta_n + 2\pi i, \theta_1, \dots, \theta_{n-1})$$



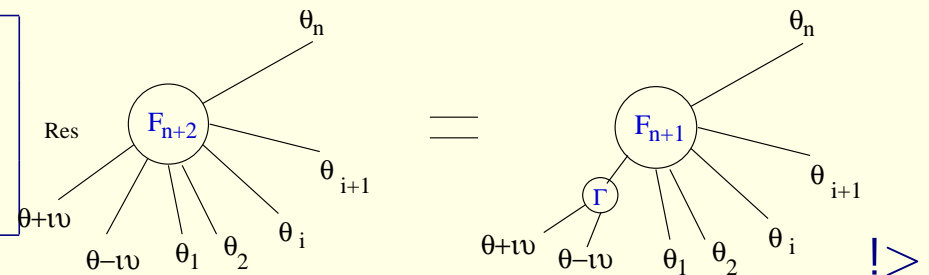
Kinematical singularities

$$-i \operatorname{res}_{\theta=\theta'} F_{n+2}^\circ(\theta + i\pi, \theta', \theta_1, \dots, \theta_n) = \left(1 - \prod_{i=1}^n S(\theta - \theta_i)\right) F_n^\circ(\theta_1, \dots, \theta_n)$$



Dynamical singularities

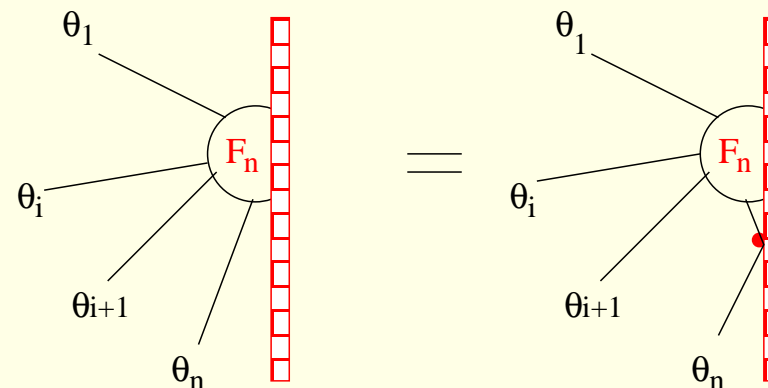
$$-i \operatorname{res}_{\theta=\theta'} F_{n+2}^\circ(\theta + iu, \theta' - iu, \theta_1, \dots, \theta_n) = \Gamma F_{n+1}^\circ(\theta, \theta_1, \dots, \theta_n)$$



Boundary form factor axioms I

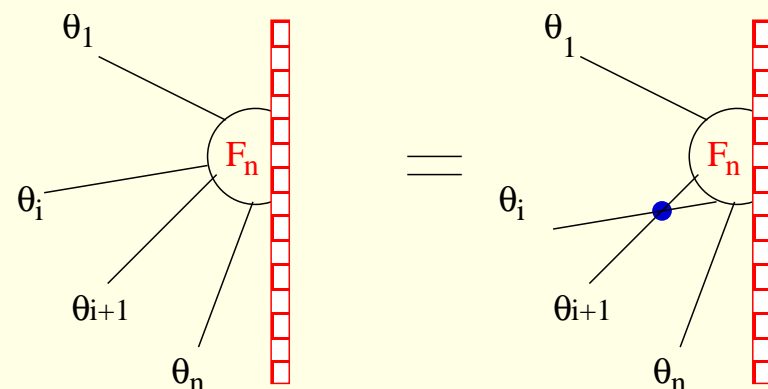
Reflection

$$F_n^{\circlearrowleft}(\theta_1, \dots, \theta_{n-1}, \theta_n) = R(\theta_n) F_n^{\circlearrowleft}(\theta_1, \dots, \theta_{n-1}, -\theta_n)$$



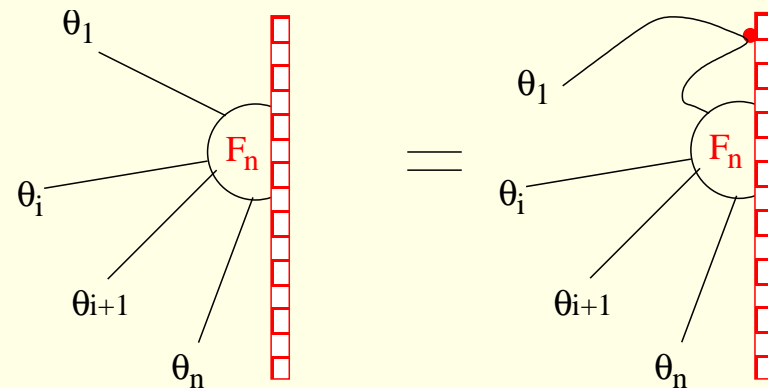
Permutation

$$F_n^{\circlearrowleft}(\theta_1, \dots, \theta_i, \theta_{i+1}, \dots, \theta_n) = S(\theta_i - \theta_{i+1}) F_n^{\circlearrowleft}(\theta_1, \dots, \theta_{i+1}, \theta_i, \dots, \theta_n)$$



Boundary periodicity

$$F_n^{\circlearrowleft}(\theta_1, \theta_2, \dots, \theta_n) = R(i\pi - \theta_1) F_n^{\circlearrowleft}(2i\pi - \theta_1, \theta_2, \dots, \theta_n)$$



Boundary form factor axioms II: kinematical singularities

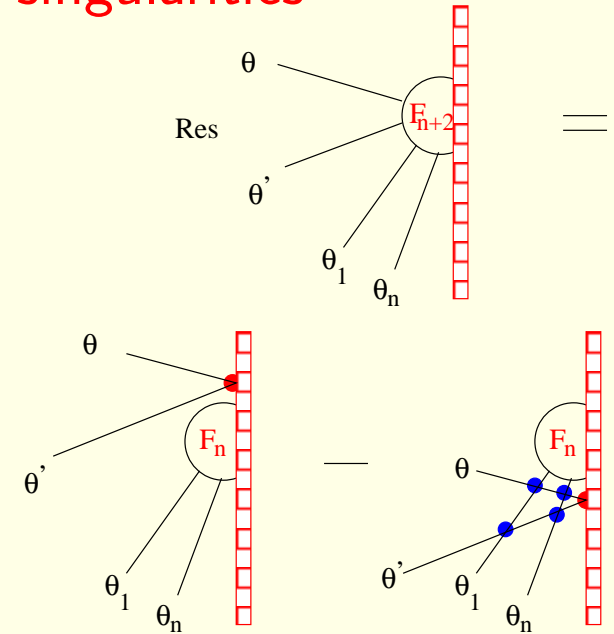
Bulk kinematical singularities

$$-i \operatorname{res}_{\theta=\theta'} F_{n+2}^{\circ}(\theta + i\pi, \theta', \theta_1, \dots, \theta_n) =$$

$$\left(1 - \prod_{i=1}^n S(\theta - \theta_i) S(\theta + \theta_i)\right) F_n^{\circ}(\theta_1, \dots, \theta_n)$$

$$-i \operatorname{res}_{\theta=\theta'} F_{n+2}^{\circ}(-\theta + i\pi, \theta', \theta_1, \dots, \theta_n) =$$

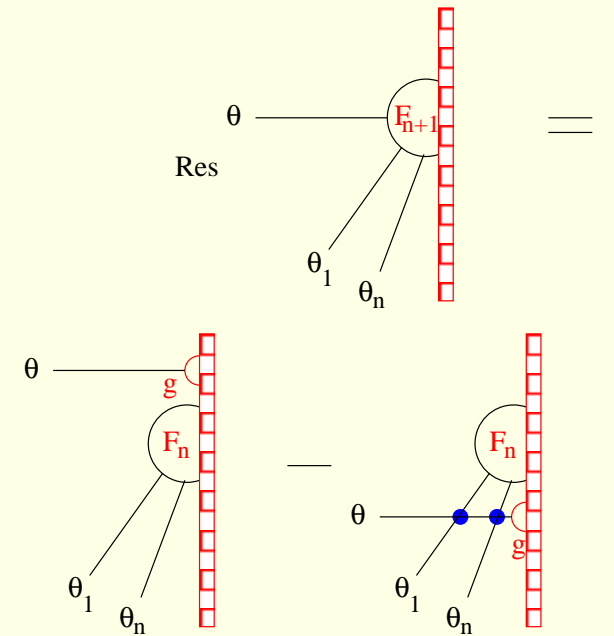
$$\left(R(\theta) - \prod_{i=1}^n S(\theta - \theta_i) R(\theta) S(\theta + \theta_i)\right) F_n^{\circ}(\theta_1, \dots, \theta_n)$$



Boundary kinematical singularities

$$-i \operatorname{res}_{\theta=i\pi/2} F_{n+1}^{\circ}(\theta, \theta_1, \dots, \theta_n) =$$

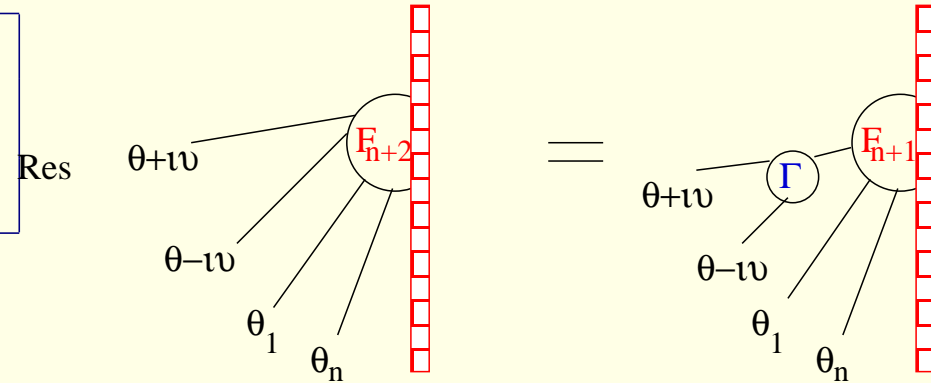
$$\frac{g}{2} \left(1 - \prod_{i=1}^n S\left(\frac{i\pi}{2} - \theta_i\right)\right) F_n^{\circ}(\theta_1, \dots, \theta_n)$$



Boundary form factor axioms III: dynamical singularities

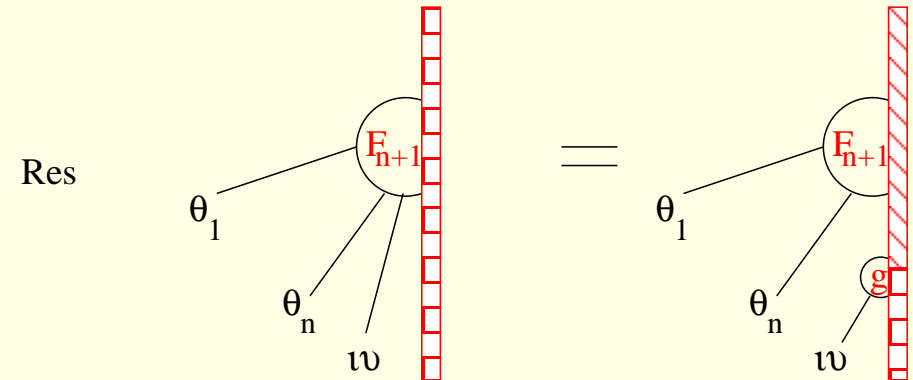
Bulk dynamical singularities

$$-i \operatorname{res}_{\theta=\theta'} F_{n+2}^{\circ}(\theta + iu, \theta' - iu, \theta_1, \dots, \theta_n) = \Gamma F_{n+1}^{\circ}(\theta, \theta_1, \dots, \theta_n)$$



Boundary dynamical singularities

$$-i \operatorname{res}_{\theta=iu} F_{n+1}^{\circ}(\theta_1, \dots, \theta_n, \theta) = \tilde{g} \tilde{F}^{\circ}(\theta_1, \dots, \theta_n)$$



!>

Solving the bulk form factor equations

Bulk theory with $S(\theta)$: FF axioms + minimality \rightarrow Solution of the FF equations

Step 1. Solve first the two particle case $F_2(\theta_1 - \theta_2) = f(\theta)$

$$f(\theta) = S(\theta)f(-\theta) \quad ; \quad f(i\pi + \theta) = f(i\pi - \theta)$$

minimality: only dynamical poles, one zero at $\theta = 0$, minimal growth at $\theta \rightarrow \infty$

Step 2. Take the Ansatz to satisfy the permutation and the periodicity axioms

$$F_n(\theta_1, \dots, \theta_n) = H_n Q_n(x_1, \dots, x_n) \prod_{i < j} f(\theta_i - \theta_j) \frac{1}{x_i + x_j} \quad ; \quad x = e^\theta$$

$Q_n(x_1, \dots, x_n)$: completely symmetric polynomial

Step 3. Derive recursion relations from the singularity axioms

$$\text{kinematical } Q_{n+2}(-x, x, x_1, \dots, x_n) = P_n(x|x_1, \dots, x_n) Q_n(x_1, \dots, x_n)$$

$$\text{dynamical } Q_{n+1}(e^{iu}x, e^{-iu}x, x_1, \dots, x_n) = R_n(x|x_1, \dots, x_n) Q_n(x, x_1, \dots, x_n)$$

Step 4. Solve recursion, classify the solutions: operator content

Perturbed Lee-Yang model

Bulk theory with $S(\theta) = \frac{\sinh \theta + i \sin \frac{\pi}{3}}{\sinh \theta - i \sin \frac{\pi}{3}}$

Step 1. Solution of the two particle case $F_2(\theta_1 - \theta_2) = f(\theta)$

$$f(\theta) = \frac{\cosh \theta - 1}{\cosh \theta + \frac{1}{2}} v(i\pi - \theta) v(-i\pi + \theta) \quad ; \quad v(\theta) = \exp \left\{ 2 \int_0^\infty \frac{dx}{x} e^{i\theta x/\pi} \frac{\sinh \frac{x}{2} \sinh \frac{x}{3} \sinh \frac{x}{6}}{\sinh^2 x} \right\}$$

minimality: dynamical pole at $\theta = \frac{2i\pi}{3}$, zero at $\theta = 0$, growth: $f(\theta) \rightarrow 1$ for $\theta \rightarrow \infty$

Step 2. The Ansatz satisfying the permutation and periodicity axioms

$$F_n(\theta_1, \dots, \theta_n) = H_n Q_n(x_1, \dots, x_n) \prod_{i < j} \frac{f(\theta_i - \theta_j)}{x_i + x_j} \quad ; \quad H_n = \langle \mathcal{O} \rangle \left(\frac{3^{1/4}}{2^{1/2} v(0)} \right)^n$$

$Q_n(x_1, \dots, x_n)$: completely symmetric polynomial

Step 3. Recursion relations from the singularity axioms

kinematical $Q_{n+2}(-x, x, x_1, \dots, x_n) = P_n(x|x_1, \dots, x_n) Q_n(x_1, \dots, x_n)$

$$P_n(x|x_1, \dots, x_n) = \frac{(-1)^n x^2}{2(x_+ - x_-)} \left(\prod_{i=1}^n (x_i + x_+) (x_i - x_-) - \prod_{i=1}^n (x_i - x_+) (x_i + x_-) \right) \quad x_\pm = e^{\mp i \frac{\pi}{3} x}$$

dynamical $Q_{n+1}(x_+, x_-, x_1, \dots, x_n) = x \prod_{i=1}^n (x + x_i) Q_n(x, x_1, \dots, x_n)$

Perturbed bulk Lee-Yang: minimal form factor solution

Step 4. Solve recursion and classify them: 2pt function and operator content

$$Q_1 = 1; \quad Q_2 = \sigma_1; \quad Q_3 = \sigma_1 \sigma_2 \dots \quad , \quad \prod_{i=1}^n (x + x_i) = \sum_{k=0}^n x^k \sigma_{n-k}$$

The solution with minimal degree is unique. Which operator is it? Compute 2pt:

$$\langle 0 | \mathcal{O}(x, 0) \mathcal{O}(0, 0) | 0 \rangle = \sum_{n=0}^{\infty} (-1)^n \int_0^{\infty} \frac{d\theta_1 \dots d\theta_n}{n! (2\pi)^n} |F_n^{\mathcal{O}}(\theta_1, \dots, \theta_n)|^2 e^{-mx \sum_{i=1}^n \cosh \theta_i}$$

Large distance expansion \leftrightarrow Compare to short distance (UV) expansion

$$\mathcal{A} = \mathcal{A}_{(2,5)} + g \int d^2x \Phi_{\left(\frac{1}{5}, \frac{1}{5}\right)}$$

$\Phi_{\left(\frac{1}{5}, \frac{1}{5}\right)} = \Phi$ field with smallest scaling dimension

$$\langle 0 | \Phi(x, 0) \Phi(0, 0) | 0 \rangle = x^{\frac{4}{5}} + x^{\frac{2}{5}} C_{\Phi\Phi}^{\Phi} \langle \Phi \rangle + \dots$$

Perturbed bulk Lee-Yang two point function

Conformal limit compared to form factor expansion

$$\langle 0 | \Phi(x, 0) \Phi(0, 0) | 0 \rangle$$

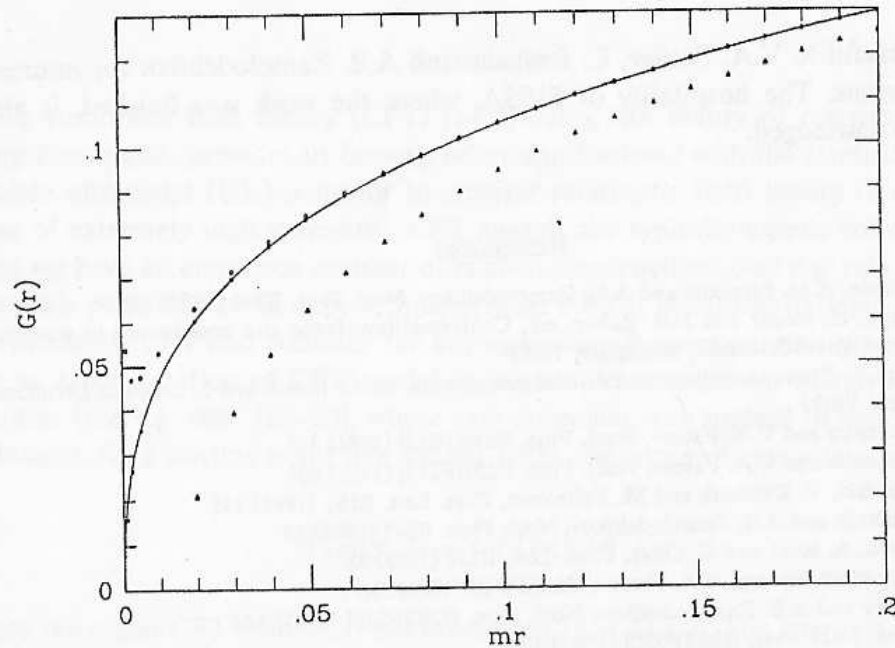


Fig. 3. Convergence of the large-distance expansion for small mr . Empty triangles: zero- and one-particle contributions. Empty circles: the same plus two-particle term. Full circles: up to three-particle state contributions. Full curve: the short-distance data.

$$\text{Conformal limit: } x^{\frac{4}{5}} + x^{\frac{2}{5}} C_{\Phi\Phi}^{\Phi} \langle \Phi \rangle$$

$$\text{FF expansion 0+1+2+3pt: } |F_0^{\Phi}|^2 +$$

$$- \int_{-\infty}^{\infty} \frac{d\theta}{2\pi} |F_1^{\Phi}|^2 e^{-mx \cosh \theta}$$

$$+ \int_{-\infty}^{\infty} \frac{d\theta_1 d\theta_2}{2(2\pi)^2} |F_2^{\Phi}(\theta_1, \theta_2)|^2 e^{-mx(\cosh \theta_1 + \cosh \theta_2)}$$

$$+ \int_{-\infty}^{\infty} \frac{d\theta_1 d\theta_2 d\theta_3}{6(2\pi)^3} |F_3^{\Phi}(\theta_1, \theta_2, \theta_3)|^2 \dots$$

Al. B. Zamolodchikov NPB348 (1991) 619.

Solving the boundary form factor equations

Boundary theory with $S(\theta), R(\theta)$: BFF axioms + minimality \rightarrow Solution of the BFF eqs

Step 1. Solve first the one particle case $F_1(\theta) = r(\theta)$

$$r(\theta) = R(\theta)r(-\theta) \quad ; \quad r(i\pi - \theta) = R(\theta)r(i\pi + \theta)$$

minimality: only dynamical poles, one zero at $\theta = 0$, minimal growth at $\theta \rightarrow \infty$

Step 2. Take the Ansatz to satisfy the reflection, permutation and the bperiodicity axioms

$$F_n(\theta_1, \dots, \theta_n) = H_n Q_n(y_1, \dots, y_n) \prod_i r(\theta_i) \frac{1}{y_i} \prod_{i < j} f(\theta_i + \theta_j) f(\theta_i - \theta_j) \frac{1}{y_i + y_j}$$

$Q_n(y_1, \dots, y_n)$: completely symmetric polynomial of $y = e^\theta + e^{-\theta}$

Step 3. Derive recursion relations from the singularity axioms

bulk kinematical $Q_{n+2}(-y, y, y_1, \dots, y_n) = P_n(y|y_1, \dots, y_n) Q_n(y_1, \dots, y_n)$

boundary kinematical $Q_{n+1}(0, y_1, \dots, y_n) = \tilde{P}_n(y_1, \dots, y_n) Q_n(y_1, \dots, y_n)$

dynamical $Q_{n+1}(y_+, y_-, y_1, \dots, y_n) = R_n(y|y_1, \dots, y_n) Q_n(y, y_1, \dots, y_n)$

Step 4. Solve recursion, classify the solutions: operator content

Perturbed boundary Lee-Yang model

Boundary theory with $S(\theta) = -\left[\frac{1}{3}\right]$, $R(\theta) = \left(\frac{1}{2}\right) \left(\frac{1}{6}\right) \left(-\frac{2}{3}\right) \left[\frac{b+1}{6}\right] \left[\frac{b-1}{6}\right]$

Step 1. Solution of the one particle case $F_1(\theta) = r(\theta)$

$$r(\theta) = \frac{i \sinh(\theta)}{(\sinh \theta + i \sin \frac{\pi}{6}(b+1))(\sinh \theta - i \sin \frac{\pi}{6}(b+1))} u(\theta)$$

$$u(\theta) = \exp \left\{ \int_0^\infty \frac{dt}{t} \left[\frac{1}{\sinh \frac{t}{2}} - 2 \cosh \frac{t}{2} \cos \left(\frac{i\pi}{2} - \theta \right) \frac{t \sinh \frac{5t}{6} + \sinh \frac{t}{2} - \sinh \frac{t}{3}}{\sinh^2 t} \right] \right\}$$

minimality: dynamical poles at $\theta = \frac{i\pi(b \pm 1)}{6}$, zero at $\theta = 0$, growth: $r(\theta) \rightarrow 1$

Step 2. The Ansatz satisfying the permutation and periodicity axioms

$$F_n(\theta_1, \dots, \theta_n) = H_n Q_n(y_1, \dots, y_n) \prod_i \frac{r(\theta_i)}{y_i} \prod_{i < j} \frac{f(\theta_i - \theta_j) f(\theta_i + \theta_j)}{y_i + y_j}; \quad H_n = \langle \mathcal{O} \rangle \left(\frac{3^{1/4}}{2^{1/2} v(0)} \right)^n$$

Step 3. Recursion relations from the singularity axioms

bulk kinematical $Q_{n+2} = P_n Q_n$ with $\beta_k(b) = 2 \cos(\frac{\pi}{6}(b+k))$ and $y_\pm = 2 \cosh(\theta \pm \frac{i\pi}{3})$

$$P_n = \frac{(y^2 - \beta_1^2(b))(y^2 - \beta_{-1}^2(b))}{2(y_+ - y_-)} \left(\prod_{i=1}^n (y_i + y_+) (y_i - y_-) - \prod_{i=1}^n (y_i - y_+) (y_i + y_-) \right)$$

dynamical $Q_{n+1}(y_+, y_-, y_1, \dots, y_n) = (y^2 + \beta_{-3}^2(b)) y \prod_{i=1}^n (y + y_i) Q_n(y, y_1, \dots, y_n)$

Perturbed boundary Lee-Yang: minimal form factor solution

Step 4. Solve recursion and classify them: 2pt function and operator content

$$Q_1 = \sigma_1; \quad Q_2 = \sigma_1(\sigma_2 + 3 - \beta_{-3}^2(b));$$

$$Q_3 = \sigma_1(\sigma_1(\sigma_2 + \beta_1^2(b))(\sigma_2 + \beta_{-1}^2(b)) - (\sigma_2 + 3)(\sigma_1\sigma_2 - \sigma_3)) \dots$$

The solution with minimal degree is unique. Which operator is it? Compute 2pt:

$$\langle 0 | \mathcal{O}(t) \mathcal{O}(0) | 0 \rangle = \sum_{n=0}^{\infty} (-)^n \int_0^{\infty} \frac{d\theta_1 \dots d\theta_n}{n! (2\pi)^n} |F_n^{\mathcal{O}}(\theta_1, \dots, \theta_n)|^2 e^{-mt \sum_{i=1}^n \cosh \theta_i}$$

Large distance expansion. \leftrightarrow Compare to short distance (UV) expansion

$$\mathcal{A} = \mathcal{A}_{BCFT(2,5)} + g \int_{-\infty}^{\infty} dt \int_{-\infty}^0 dx \Phi_{(\frac{1}{5}, \frac{1}{5})}(x, t) + h \int_{-\infty}^{\infty} dt \varphi_{\frac{1}{5}}(t)$$

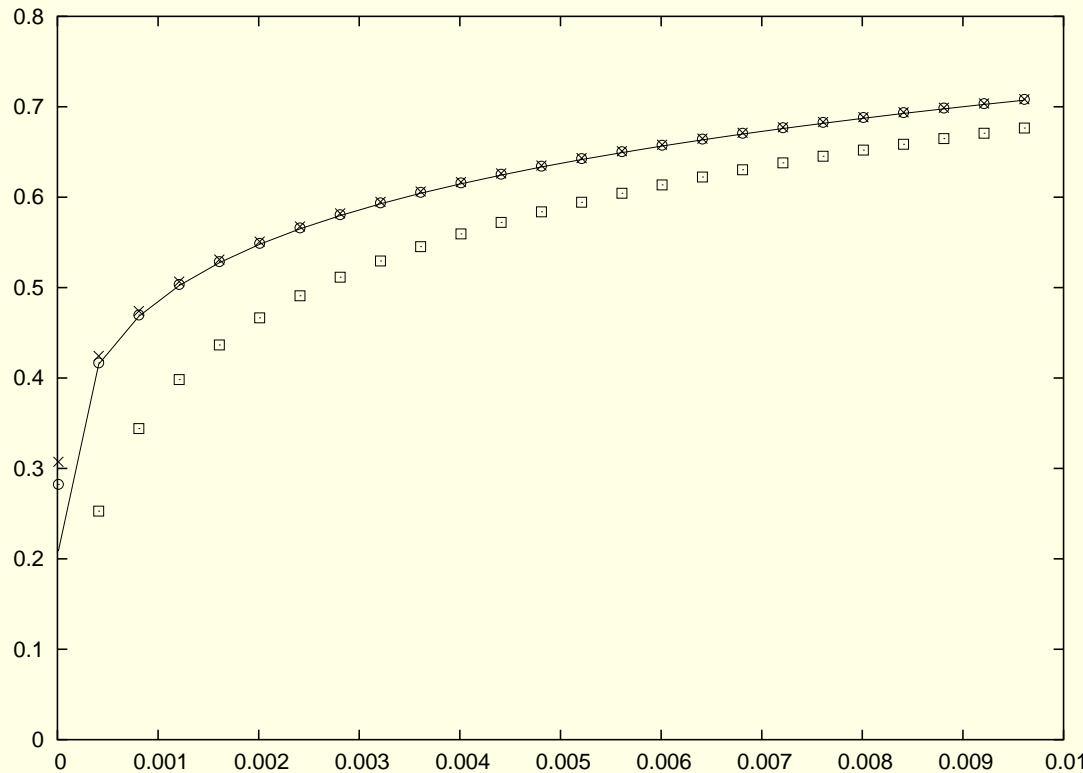
$\varphi_{\frac{1}{5}} = \varphi$ boundary field with smallest scaling dimension

$$\langle 0 | \varphi(t) \varphi(0) | 0 \rangle = -t^{\frac{2}{5}} + t^{\frac{1}{5}} C_{\varphi\varphi}^{\varphi} \langle \varphi \rangle + \dots$$

Perturbed boundar Lee-Yang two point function

Conformal limit compared to form factor expansion

$$\langle 0 | \varphi(t) \varphi(0) | 0 \rangle$$



Conformal limit: $-t^{\frac{2}{5}} + t^{\frac{1}{5}} C_{\varphi\varphi}^{\varphi} \langle \varphi \rangle$

BFF expansion 0+1+2+3pt: $|F_0^{\varphi}|^2 +$

$$- \int_0^{\infty} \frac{d\theta}{2\pi} |F_1^{\varphi}|^2 e^{-mt \cosh \theta}$$

$$+ \int_0^{\infty} \frac{d\theta_1 d\theta_2}{2(2\pi)^2} |F_2^{\varphi}(\theta_1, \theta_2)|^2 \cdot e^{-mt(\cosh \theta_1 + \cosh \theta_2)}$$

$$+ \int_0^{\infty} \frac{d\theta_1 d\theta_2 d\theta_3}{6(2\pi)^3} |F_3^{\varphi}(\theta_1, \theta_2, \theta_3)|^2 \dots$$

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