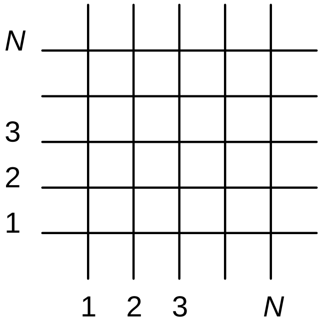
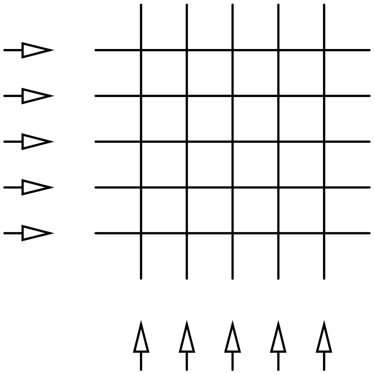
Domain wall partition functions, plane partitions and free fermions

Joint work with A Caradoc, A Dow, N Kitanine, M Wheeler and M Zuparic

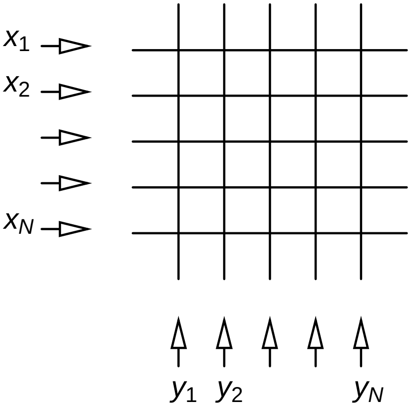
U of Melbourne + U de Cergy-Pontoise



Our playing field



The lines have orientations.


x_1
x_2
x_N
y_1
y_2
y_N

The lines intersect at intersection points.

A segment between two adjacent intersection points is a bond.

On every bond, we put a state variable.

In the six vertex model, the state variable is an arrow that can point in either direction

An intersection point, with four adjacent bonds, with state variables on them, is a vertex.

To each vertex we assign a weight.

The weight of a vertex is a function of the state variables and the rapidity variables that flow through the intersection point.

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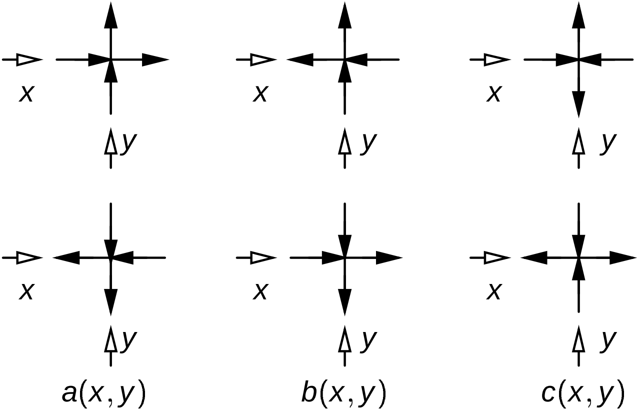
x
y
x
y
x
y
x
y
x
y
x
y
a(x, y)
b(x, y)
c(x, y)

$[x] = \sinh(\lambda x)$, λ is the crossing parameter.

$a(x_i, y_j) = [-x_i + y_j + 1]$

$b(x_i, y_j) = [-x_i + y_j]$

$c(x_i, y_j) = [1]$

The weights satisfy the Yang-Baxter equation.

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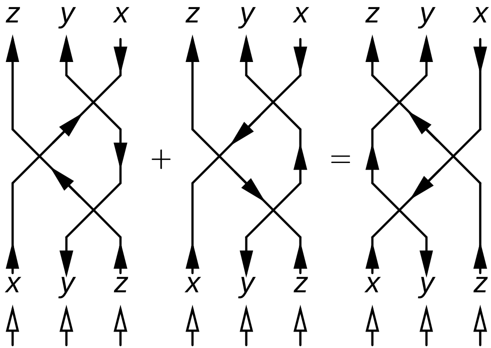
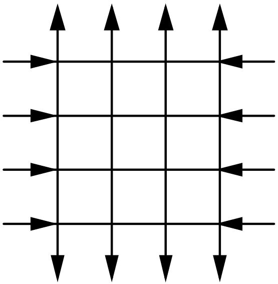
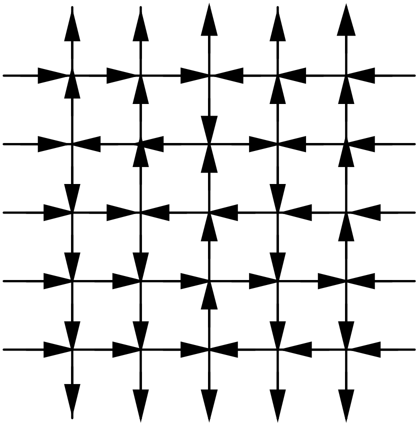

z y x
x y z
+
z y x
x y z
=
z y x
x y z





$$Z_N\left(\{x\},\{y\}\right) = \sum_{\substack{\text{all allowed}\\ \text{configurations}}} \prod_{\text{vertices}} w_{ij}$$

Izergin's determinant expression

$$Z_N\left(\{x\},\{y\}\right) = \frac{\prod_{i,j=1}^{N}[-x_i+y_j+1][-x_i+y_j]}{\prod_{1\le i<j\le N}[-x_i+x_j][-y_j+y_i]}\,\mathrm{det}\left(M\right)$$

$$M_{ij} = \frac{[1]}{[-x_i+y_j+1][-x_i+y_j]}$$

Correlation functions

Alternating sign matrices

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Alternating sign matrices

QUANTUM INVERSE SCATTERING METHOD AND CORRELATION FUNCTIONS

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$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad \begin{pmatrix} 0 & 1 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \qquad \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix},$$

$$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \qquad \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$